Financial Crises and Endogenous Volatility*

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Abstract

During the recent U.S. financial crisis, initial losses in financial markets were accompanied by a widespread increase in returns volatility of non-financial firms. Motivated by this feature, I develop a novel mechanism in which firms' volatility arises endogenously as a result of financial disruptions. In the model, firms have access to a set of different projects, some of which have a higher expected return, but also a higher variance. Firms can issue debt and, because they have the option to default, the cost of borrowing increases with their project riskiness selection. As a result, periods of high borrowing incentivize the firm to undertake low risk-projects, while firms take on more risk during periods of low borrowing. Consequently, financial shocks that initially lead to a reduction in firms' borrowing, induce an increase in volatility through firms' project selection. The model can endogenously generate the increase in equity returns volatility witnessed during the 2007-2009 recession, as well as replicate the contractions observed in output, investment and hours worked. The endogenous volatility mechanism accounts for approximately 65% of the decline in these variables. I provide micro-evidence that supports the key model implications.

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1 Introduction

A line of recent research has argued that volatility is countercyclical and has harmful economic implications.[1] However, less work has been done in explaining the source and magnitude of volatility fluctuations.[2] In this paper I develop a dynamic general equilibrium model in which firm level volatility endogenously arises as a result of first moment shocks to a financial sector. As in much of the recent work, I place a particular emphasis on the 2007-2009 recession in which volatility markedly increased. Quantitatively, the model can endogenously generate the increase in volatility witnessed over the Great Recession as well as the contraction in output, investment and hours. In this regard, the endogenous volatility mechanism is key and accounts for roughly 65% of the decline in these variables.

As previous work documented, measures of volatility rise during recessions. Figure 1 illustrates this with the cross-sectional volatility of equity returns for non-financial firms in the U.S.[3] Although fluctuations in volatility could be a cause of recessions, it is also reasonable to entertain the hypothesis of reverse causality. This is particularly true for the 2007-2009 crisis, which is commonly believed to have originated with large losses in the financial sector and occurred simultaneously with the largest historical increase in equity returns volatility. Figure 2 shows the market value of financial firms and equity returns volatility around the Great Recession. Motivated by these observations, I build a model that can successfully generate large fluctuations in volatility as a response to alterations in financial conditions, in line with the sequence of observed events.

The key new assumption in the model is the firm's ability to determine her project's riskiness. Firms have access to a set of different projects, some of which have a higher expected return, but also a higher variance. Firms are risk-neutral and thus prefer the project with the highest expected return. In order to fund this project (i.e., purchase capital) firms can issue defaultable debt. Because the firm has an option to default, the cost of borrowing increases with the riskiness

[1] See Bloom (2014) for a recent survey.

[2] I discuss previous work done on this in Section 2.

[3] The correlation between GDP growth and equity returns volatility is -0.24. Returns are obtained from CRSP on a quarterly basis and include dividend payments. Volatility corresponds to cross-sectional variation of returns across firms in a given quarter. Appendix A details data construction and Appendix C elaborates on the volatility computations.

of the project she selects.[4] Consequently, the firm faces a trade-off: she can select more profitable and riskier projects, but at the expense of increasing her cost of borrowing. This trade-off crucially depends on the amount of debt that the firm is selling: during periods of high borrowing, the firm has incentives to undertake low-risk projects and obtain cheap funding. On the other hand, during periods of low borrowing, the firm finds it profitable to move towards riskier projects. As a result, periods of decreasing borrowing are associated with increasing profits volatility.

I incorporate this trade-off in a model of financial intermediation as in Gertler and Kiyotaki (2009). In this environment, banks' capacity to lend is constrained by their net worth. As a result, banks decrease lending during periods of capital losses, which triggers the firm's riskiness decision described above: firms move towards riskier projects and their profits volatility rises. Consequently, first moment shocks to banks' net worth induce second moment responses in firms' returns, in line with the events observed during the Great Recession. Furthermore, I show that the endogenous increase in volatility greatly amplifies the effects of the initial financial disruption.

To my knowledge, this is the first model that successfully explains the increase in equity returns volatility during the last recession. Core to this result is the mean-variance technological relation assumed at the firm level.[5] In the quantitative evaluation of the model, I discipline this technological assumption by matching certain empirical features of equity returns volatility. I demonstrate how fluctuations in financial conditions trigger firms' volatility, which in turn amplifies the initial financial disruptions. I show that financial shocks that induce a decline in financial firms' market value as observed in Figure 2, are quantitatively consistent with the observed increase in equity returns during the 2007-2009 crisis. The model is also successful in explaining the drop in investment during the 2007-2009 crisis. In this regard, the endogenous volatility mechanism is key and accounts for 66% of the decline in investment.

However, as in many macro-finance models, financial shocks in my model affect employment and output only mildly.[6] In order to overcome this issue, I add frictions to the labor market that

[4]This is an equilibrium outcome. The basic intuition is that, because the firm can default, bondholders absorb the downside risk of firm's profits realization. An increase in the firm's profits riskiness induces bondholders to ask for a higher premium. See Merton (1974) for an earlier discussion.

[5]In Section 3, I discuss how this technological assumption relates to the firms' risk-management literature. Appendix E provides micro-foundations for these assumptions.

[6]See Section 5.2 for a further discussion.

link financial conditions and hiring costs.[7] In particular, I extend the model to incorporate sticky wages and a working capital constraint. Once these frictions are added, I show that the model can also match the drop in output and hours observed during the 2007-2009 crisis. The endogenous volatility mechanism remains crucial and explains about 65% of the drop in output and hours worked.

I provide micro-evidence that supports the key implications of the model. In particular, I show that equity returns volatility of publicly traded firms in the U.S. are consistently lower for firms with higher debt issuance. This is in line with the implications of the model, in which firms' incentives to take on riskier projects increases during periods of low borrowing. Furthermore, I argue that during the Great Recession, equity returns volatility increased remarkably more for firms that decreased borrowing the most. I think of these facts as supporting evidence of the model.

The rest of the paper is organized as follows. Section 2 discusses research related to this paper. Section 3 outlines the model. Section 4 characterizes the model outcomes and provides intuition on the model mechanisms. Section 5 performs the quantitative evaluation and confronts the models with the events of the 2007-2009 recession. Section 6 provides micro-evidence for the model mechanism. Section 7 concludes and suggests extensions for future research.

2 Literature Review

My paper is related to two lines of research: (i) macroeconomic models with financial frictions; and (ii) macroeconomic models with time-varying volatility.[8] I discuss how my paper relates to each topic and comment as well on other works in the intersection of these topics.

Following Bernanke and Gertler (1989), a large body of work has been devoted to understanding the relationship between financial markets and overall macroeconomic performance. Two canonical references are Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999).[9] In these

[7] Similar assumptions are made in Jermann and Quadrini (2012) and Bigio (2013), among others. See also Blanco and Navarro (2014) and Monacelli, Quadrini, and Trigari (2011) for papers that provide a micro-founded link between financial and labor markets using search frictions.

[8] As I discuss below, there is a difference between uncertainty and volatility which hasn't always been clearly distinguished in the literature. I use time-varying volatility to refer to both of these.

[9] For more recent contributions, see: Adrian and Boyarchenko (2012), Bigio (2012), Brunnermeier and Sannikov (2012), Jermann and Quadrini (2012), Khan, Senga, and Thomas (2014), Kiyotaki and Moore (2012), Moreira and

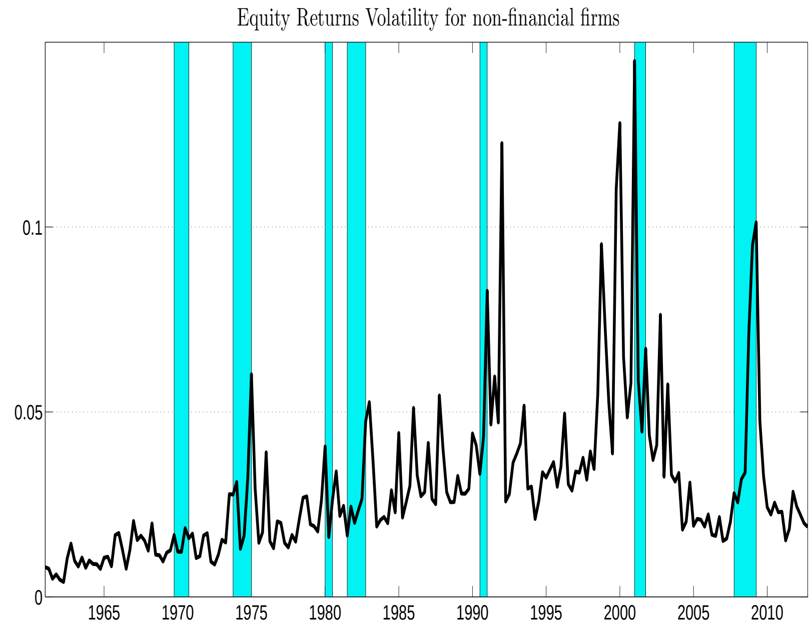


Figure 1: Cross-Sectional Dispersion of Equity Returns

Notes: *Shaded areas correspond to the recession dates as dated by the NBER. Equity returns are from CRSP for non-financial firms. Computations made by the author. See Appendix A for data sources and Appendix C* for computations.

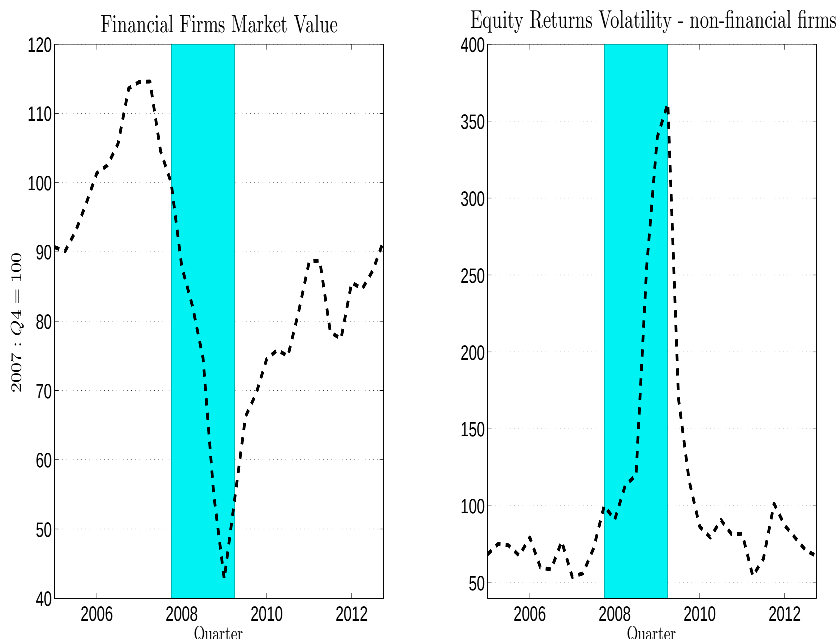


Figure 2: The 2007-2009 Crisis

Notes: *Market value of financial firms is computed as the sum of outstanding shares times the price of the shares. Data is from CRSP. Computations made by the author. See Appendix A for data details.*

papers, firms' financial position affects their cost of external funding and thus their investment decisions. After the 2007-2009 crisis, more focus has been placed on the role of financial intermediaries. The recent contributions of Gertler and Karadi (2011) and Gertler and Kiyotaki (2009) provide a canonical framework to analyze frictional financial intermediation.

I extend the work of Gertler and Kiyotaki (2009) by incorporating default risk in the non-financial sector of the economy. As typically done in the sovereign default literature (Eaton and Gersovitz, 1981, Arellano, 2008), or more recently in the macro-finance literature (Hennessy and Whited, 2007, Gomes and Schmid, 2010, Khan, Senga, and Thomas, 2014), I model default as a dynamic, forward-looking decision, where default occurs in equilibrium. This is important for two reasons. First, default risk in the non-financial sector is a key piece of the endogenous volatility mechanism. A firm's project riskiness affects the price of her debt just because she can default in equilibrium, which generates a trade-off to the firm that results in endogenous fluctuations of volatility. Second, default risk in the non-financial sector is essential for the transmission of financial shocks into real activity. I model financial shocks as an innovation that affects banks' net worth but does not affect firms' technology.[10] However, the decline in lending induces a firm to run riskier projects, which increases her default rates and induces a contraction in economic activity. Thus, the default risk in the non-financial sector, interacting with the endogenous volatility mechanism, greatly amplifies the initial financial disruptions.

Since the seminal paper by Bloom (2009), there has been a renewed interest in models with time-varying volatility.[11] Earlier theoretical contributions by Bernanke (1983) and McDonald and Siegel (1986), and recently continued by Bloom, Floetotto, Saporta-Eksten, and Terry (2012), argue that increases in uncertainty interacted with non-smooth adjustment costs can lead to economic recessions. The basic idea is that, in presence of higher uncertainty, firms have an option to delay investment, save the adjustment costs, and wait for a more promising environment. This "wait-and-see" policy generates a "bust" and a subsequent "boom" when uncertainty dissipates, an empirical pattern found by Bloom (2009). However, recent work by Bachmann and Bayer (2013)

Savov (2013) and Shourideh and Zetlin-Jones (2012) among others.

[10]See Bocola (2014) for similar assumptions in the context of sovereigns default risk.

[11]The literature in this topic is large and still growing, and I do not attempt to make a complete review in here. See Bloom (2014) and Bloom and Fernandez-Villaverde (2014) for a more comprehensive discussion.

argues that this “bust-boom” uncertaity induced cycle is unlikely to be a major source of business cycle fluctuations. In related work, Bachmann, Elstner, and Sims (2013) use several measures of volatility for the U.S. to argue that, in contrast with the findings in Bloom (2009), it is more likely that increases in uncertainty induce a negative long lasting effect on real activity. They conclude that models of time-varying volatility should be interacted with components other than adjustment cost, such as I do in this paper.

My paper is more closely related to the recent literature that incorporates time-varying volatility in dynamic models with financial frictions. Recent work by Christiano, Motto, and Rostagno (2014) extends a standard dynamic general equilibrium model as in Bernanke, Gertler, and Gilchrist (1999) to argue that fluctuations in the cross-sectional volatility of entrepreneurial returns are the most important driver of business cycle fluctuations for the U.S. economy. In a recent paper, Gilchrist, Sim, and Zakrajsek (2013) incorporate default risk in a model of firms investment dynamics, to argue that a “wait-and-see” effect interacted with financial frictions induce responses of investment to volatility shocks that are in line with the evidence. In the same line, Arellano, Bai, and Kehoe (2012) develop a model in which hiring labor is a risky endeavour, to argue that an increase in volatility can induce a large drop in economic activity. Other examples in the literature that include stochastic volatility in models with financial frictions are Chugh (2014) and Dorofeenko, Lee, and Salyer (2008).

My contribution to this literature is to endogenize fluctuations in volatility as a result of financial conditions. In common with all of these papers, I share that frictional financial markets crucially propagate the effects of fluctuations in volatility. In addition, in this paper, financial markets are also the origin of fluctuations in volatility. I should emphasize that I do not rule out autonomous fluctuations in volatility. However, as argued above, I think that a certain degree of endogeneity in volatility fits better the sequence of events observed during 2007-2009 crisis.

This paper clearly intersects with work that treats riskiness (volatility) as outcomes of endogenous choices. This has a long tradition in the banking literature, where riskiness is typically measured by the quality of loans that banks give. See Allen and Gale (2000) for a discussion, or Boyd and De Nicolo (2005) more recently, and Chari and Kehoe (2008) for a discussion in the context of a monetary union. Similar logic is used in Begenau (2014), who studies risk choice at a bank level and its interaction with capital requirement constraints. As in my model, Corbae and D'Erasmo (2013) allow for risk choices at the firm level in a quantitative model of banking. A micro-foundation of firms risk based in market exposure is given in Decker, D'Erasmo, and Moscoso Boedo (2014). Finally, Kehrig (2011) provides very intersting data on the source of productivity dispersion, which is continued in the work of Ilut, Kehrig, and Schneider (2014).

Closest to my work is Tian (2013) who develops a model of firm dynamics with endogenous exit. As in my model, the exit option adds convexity to the firms' value function, which makes an increase in variance desirable for firms close to exiting: an "option value" effect.[12] There are two main differences between our papers. First, I focus on the effect of financial, instead of productivity, shocks. Second, and more important, I construct an equilibrium in which firms' borrowing costs reflect their riskiness choice. I find that this dominates the "option value" effect and completely eliminates firms' incentives to increase volatility.[13] This is the reason why I introduce a productivity premium for risk-taking in the model. I believe that allowing for borrowing costs to depend on firms' riskiness decision is a better description of financial intermediation, a core theme in my paper.

Another line of work endogenizes time-varying volatility as the result of learning about uncertain environments. A seminal contribution is Veldkamp (2005), which has recently been extended by Orlik and Veldkamp (2014), Fajgelbaum, Taschereau-Dumouchel, and Schaal (2014) and Bachmann and Moscarini (2012) among others.

3 Model

Time is discrete an indexed by $t = 0, 1, 2, \ldots$. The economy is populated by firms, banks, capital producers, and a representative household. Firms have access to a production technology given as

$$y = (xak)^{\alpha} n^{1-\alpha} \tag{1}$$

[12]See Jensen and Meckling (1976) for an earlier discussion on this.

[13]See Appendix F for a more detailed discussion.

where k and n stand for firm's capital and labor, respectively. Firms' productivity has an aggregate component x, which is identical for all firms, and an idiosyncratic component a. While aggregate productivity x follows a given process, firms can choose the distribution of the idiosyncratic productivity a from a set of available functions. This is a key new assumption in the model and is discussed in more detail below.

In order to fund her activities, firms can issue debt or equity.[14] Since firms may decide to default on their liabilities, debt is risky. I further assume that firms' debt has long maturity: in order to avoid default, a firm must pay a coupon c plus a fraction λ of her total outstanding liabilities b. This form of modeling long-term debt is convenient because of its tractability.[15] Upon default, the firm exits the economy and a fraction Ξ of her capital gets destroyed.[16]

Banks are the other interesting agent in financial markets. Every period, banks can invest in firms' bonds as well as in a risky security. The risky security is a financial asset to which only banks have access to, and pays a stochastic yield ξ every period.[17] I think of the return to the risky security ξ as the financial shock in the economy, because it directly affects banks' net worth.[18] In order to finance her portfolio, banks can use their own net worth as well as take deposits from households. However, following Gertler and Kiyotaki (2009), I assume a simple agency problem that limits banks' capacity to intermediate as a function of banks' net worth. The main contribution of this paper is to show how banks' net worth losses result in both a decline in lending and an endogenous increase in firms' profits volatility.

As mentioned above, the key new element in the model is firms' ability to chose the distribution of her idiosyncratic productivity a. I assume that firms can choose between projects that have a higher expected return, but also a higher volatility. In particular, I make the following assumption.

[14] In this environment, I understand equity injections as negative dividend payments. Similar assumptions are made in Blanco and Navarro (2014) and Gomes and Schmid (2010) among others.

[15] See Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009) among others. The coupon payments c is treated as a parameter, similar modeling assumptions can be found in Gomes, Jermann, and Schmid (2013). As in all of these papers, I think of $1/\lambda$ as the average maturity of the debt.

[16] Since firms have constant returns to scale, the number of firms is not important. In order to keep the mass of firms strictly positive, one could simply assume that randomly some firms are split into two and equally divide assets and liabilities.

[17] This risky security can be thought of as a "Lucas tree", see Lucas (1978).

[18] In Section 5, I calibrate the risky security to resemble the size of the real estate market in the U.S.

Assumption 1 *Every period, firms can* ***choose*** *the distribution of the idiosyncratic productivity from a family of log-normal distributions:* $\ln a \sim \mathcal{N}\left(\mu(\sigma), \sigma^2\right)$. *The function* $\mu(\sigma)$ *is given as* $\mu(\sigma) = \mu_a + (\varphi_1 - \varphi_2\sigma)\,\sigma$, *with* $\varphi_1, \varphi_2 > 0$. *Thus,* $\mu(\sigma)$ *is concave and a unique level* $\bar{\sigma}$ *exists such that* $\bar{\sigma} = \arg\max_\sigma \{\mu(\sigma)\}$. *Finally,* σ *must be chosen one period in advanced.*

Assumption 1 is the key innovation in the model and a few comments are in order. First, firms' σ choice will be a major determinant of default probabilities, and I will refer to this as the firms' **risk** choice. As argued above, bad financial conditions will increase firms' **risk** choice σ which will become countercyclical, in line with the evidence. Second, note that the idiosyncratic productivity a is *independent* across firms and time. This will simplify aggregation at the firm level later on.[19] Third, firms face a mean-variance project selection: for initial low levels of σ, a firm can increase her expected returns at the cost of increasing her variance. Although I take this mean-variance relation as a technological description of the economy, it can be interpreted as the result of the firm's risk-management decision. For instance, as I show in Appendix E, this mean-variance relation can be obtained in an environment where firms have the possibility to hedge against stochastic fluctuations in the cost of an input.[20]

In my model, the increase in firms' productivity is necessary to induce firms to take on risk. As initially argued by Jensen and Meckling (1976), the default option adds convexity to the firm's value function which makes increases in σ desirable to her. This is sometimes referred as an option value effect. A key novelty in my model is that the price of the firm's debt is a function of her risk choice σ, which disincentives her to take on risk. I found that this latter effect dominates the option value effect in my model, and thus I needed to assume an increasing function $\mu(\sigma)$ to induce a trade-off at the firm level.[21] Imposing a value of risk $\bar{\sigma}$ that maximizes firms returns is simply to ensure interior solutions.

Timing within a period is as follows. At the beginning of period t, every firm starts with an

[19]This type of assumption has extensively been used in the literature of macroeconomic models with financial frictions. See for instance Gertler and Kiyotaki (2009), Kiyotaki and Moore (2012) and Bigio (2012) where they assume this for investment opportunities.

[20]The example in Appendix E borrows from Froot, Scharfstein, and Stein (1993), whose pioneer paper guided much of the theoretical and empirical work in firms' risk-management. See Stulz (1996) for an empirical discussion, and Rampini, Sufi, and Viswanathan (2014) and Rampini and Viswanathan (2010) for recent contributions.

[21]In Appendix F, I develop a one period example to explain how the option value effect is dominated by the effect of σ on prices.

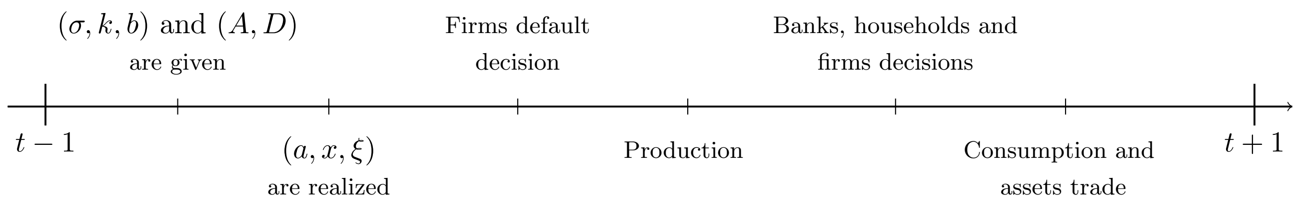


Figure 3: Timing within a period

amount of capital k, total debt b, and a previous choice of σ. Similarly, banks start with holdings of the risky security A, deposits they borrowed from households D, and a portfolio of firms' bonds. Then, the exogenous shocks arrive: the aggregate productivity x is realized, every firm learns her idiosyncratic productivity a and the return on the risky security ξ is observed as well. After this, firms decide whether to default on their debt or not. In case of repayment, she can hire labor and produce output, issue new debt, invest in new capital, make a new risk choice and pay dividends. Simultaneously, banks collect the return on previous investments, and make new portfolio decisions. Households make consumption/saving decisions and supply labor. At the end of the period every market must clear and period $t+1$ begins. Figure 3 shows the timing just described.

I will next formally describe the firms', banks' and household's problems. For the moment, I will denote S as the aggregate state of the economy and define it explicitly later on.[22]

Notation 1 *Let S denote the aggregate state of the economy and Γ its law of motion: $S' \sim \Gamma(S)$.*

3.1 Firms

The firms' problem contains risk choice, the new element in this paper. It also exhibits how this risk choice interacts with the firm's leverage decision, as well as default probabilities are determined.

After observing the aggregate shocks and her idiosyncratic productivity a, a firm can decide whether to default or not. If a firm doesn't default, she can hire labor n and combine it with her capital k to produce output. This is a static problem and given as follows

$$\Pi(a, k, S) = \max_n \left\{ (axk)^\alpha n^{1-\alpha} - w(S)n \right\} \tag{2}$$

[22] "Finding the state is an art".

where $w(S)$ is the wage when the aggregate state of the economy is S. It is easy to verify that firm's profits $\Pi(a, k, S)$ are linear in capital. The following lemma formalizes this.

Lemma 1 ***(Firm's Profits)*** *Firm's profits $\Pi(a, k, S)$ are linear in capital and given by*

$$\Pi(a, k, S) = ak\pi(S) \tag{3}$$

where $\pi(S) = xw^{-\frac{1-\alpha}{\alpha}}\Theta$ and $\Theta = (1-\alpha)^{\frac{1-\alpha}{\alpha}} - (1-\alpha)^{\frac{1}{\alpha}}$

(All proofs are in Appendix B) Lemma 1 is a direct implication from constant returns to scale. The value $\pi(S)$ is the firms' average returns on capital. Naturally, a firm with a higher idiosyncratic productivity a will experience a larger return. The linearity of firms' profits in capital will greatly simplify the model solution.[23] I will from now onwards use $\pi(S)$ to refer to firms' profits.

After production, firms can pay dividends d, choose next period total debt b', purchase capital k' and make a risk choice σ'. Let $E(a, b, k, S)$ be the value of a *non-defaulting* firm with productivity a, total debt b and capital k when the aggregate state of the economy is S. Then

$$\begin{aligned} E(a,b,k,S) &= \max_{d,b',k',\sigma'} \left\{d + \mathbb{E}_{a',S'}\left[m(S,S')\max\left\{0, E(a',b',k',S')\right\}|S,\sigma'\right]\right\} \qquad (4) \\ &\text{subject to} \\ \mathcal{P} &= (1-\tau)\left[ak\pi(S) - cb\right] - \lambda b \\ d + Q^k(S)k' &\leq \mathcal{P} + Q^k(S)(1-\delta)k + Q(l',\sigma',S)\left[b' - (1-\lambda)b\right] \end{aligned}$$

where $l' = b'/k'$ is next period firm's leverage.

The first line in the firm's feasible set defines $\mathcal{P}$, the cash-flow available to a *non-defaulting* firm: it accounts for returns on production $ak\pi(S)$, minus coupon debt payments cb and maturing debt payments λb. Notice that the firm faces a tax τ on profits, from which debt coupon payments are deductible. This tax benefit on coupon payments induces the firm to issue debt.[24] The second line

[23] As discussed in more detail below, the combination of constant returns to scale and *independent* idiosyncratic productivity over time delivers aggregation across firms. For a discussion where the *independence* assumption is relaxed see Gomes and Schmid (2010). For a discussion where also the constant returns to scale assumption is relaxed, see Khan, Senga, and Thomas (2014).

[24] Defaults by itself breaks Modigliani-Miller irrelevance result. In particular, the firm would strictly prefer to use

in the feasible set is the firm's budget constraint: her expenditures are given by dividend payments d and new capital purchases $Q^k(S)k'$. Her available resources are the cash-flow $\mathcal{P}$, the value of non-depreciated capital $Q^k(S)(1-\delta)k$ plus the new debt issuance $[b' - (1-\lambda)b]$ times the price of the debt $Q(l', \sigma', S)$.

The firm's problem is to choose dividend payments d, next period capital k', debt b' and risk σ' in order to maximize the present discount value of her dividend payments. In doing so, she takes into account the option to default at the beginning of any period. The stream of dividend payments is evaluated using the stochastic discount factor of the household $m(S, S')$ who is the owner of the firms. I show below that this problem has a very tractable solution.

The firm can issue debt at a price $Q(l', \sigma', S)$ to finance her expenses. The price of the debt depends on the firm's leverage l' as well as on her risk decision σ'. The reason why the debt price depends on leverage and risk is because the probability of default depend on these, and bond prices reflect the likelihood of such an scenario.[25] Firms understand this and internalize it when making decisions. In Section 4, I explain in more detail how firms incorporate the effect that their decisions have on prices when computing optimal policies. Finally, note that a firm may desire to issue new debt ($b' > (1-\lambda)b$) or to decrease her liabilities ($b' < (1-\lambda)b$), in which I assume that the firm repurchases old debt.[26] Notice that because of the *independence* distribution assumption on the idiosyncratic shock, the likelihood of default next period does not depend on the current *idiosyncratic* productivity and thus neither does the price of debt.

The risk choice σ' appears in two places in the firm's problem. First, it determines the distribution of the idiosyncratic productivity a' and thus affects the expected value of the firm next period. Second, it also affects the price $Q(l', \sigma', S)$ at which the firm sells new debt and thus alters the firm's budget constraint. Then, the effect that σ' has on bond price matters to the extent that the firm is actually issuing new debt. For instance, in a case of no new debt issuance ($b' = (1-\lambda)b$), the

equity only. Thus, the tax benefit is necessary for having debt issuance in equilibrium. See Gomes, Jermann, and Schmid (2013) for similar assumptions.

[25] Similar logic can be found in models of default like Arellano (2008), Eaton and Gersovitz (1981), Gomes and Schmid (2010) or Arellano, Bai, and Kehoe (2012) among many others.

[26] An interesting extension would be to allow the firm to save in a risk-free asset. My conjecture is that, as long as the risk-free asset cannot be seized upon default, not much would change. However, given the recent path of riskless assets accumulation in firms (see Sanchez and Yurdagul, 2013), I believe this is a promising path for future research.

effect of risk σ' on debt prices becomes irrelevant to the firm. On the contrary, in periods of large new debt issuance, the firm cares more about the price of her debt. Because in equilibrium bond prices will be decreasing in σ', the logic just described implies that financial shocks that reduce lending will incentivize the firm to increase her risk choice σ'. This is precisely the new mechanism proposed in the paper.

Notice that the asset structure of this economy is extremely large. In particular, the firm is choosing to issue debt from a continuum of bonds indexed by the couple (l', σ') and with a corresponding price $Q(l', \sigma', S)$. Although firms only issue one type of debt, the entire price schedule is relevant for her decision and must then be computed. The bond price function will be obtained from banks problem, such that all bond markets indexed by (l', σ') clear in equilibrium.[27]

The following proposition characterizes firm's optimal policies

Proposition 1 ***(Firm's Optimal Policies)*** *Firm's value function is linear in capital and given as*

$$E(a, b, k, S) = \mathcal{P} + e(l, S)k \tag{5}$$

Policies for capital and debt next period are also linear in capital and given by

$$\begin{aligned} k'(a, b, k, S) &= \boldsymbol{\iota}(l, S)k \\ b'(a, b, k, S) &= \boldsymbol{\ell}'(l, S)k'(a, b, k, S) \end{aligned}$$

Similarly, optimal risk choice for next period σ' is identical across firms

$$\sigma'(a, b, k, S) = \boldsymbol{\sigma}'(l, S)$$

[27]Alternatively, one could assume that firms face a given price schedule $Q(l', \sigma', S)$, which in equilibrium must leave lenders indifferent with respect to investing in some other asset (or other bond). This is the modeling assumption typically used in the sovereign default literature, see for instance Arellano (2008). Both structures are analogous. However, multiplicity of equilibrium may arise in these environments, see Navarro, Nicolini, and Teles (2014) for a discussion.

Default follows a threshold decision: only firms with idiosyncratic productivity below a threshold $\underline{\boldsymbol{a}}(l, S)$ *will decide to default. Finally, the function* $e(l, S)$ *in equilibrium satisfies*

$$e(l, S) = Q^k(S)(1 - \delta) - (1 - \lambda)Q(\boldsymbol{\ell}'(l, S), \boldsymbol{\sigma}'(l, S), S)l \tag{6}$$

Proposition 1 shows that the firm problem has a simple solution: per unit of capital, every firm is making the same investment and debt decisions. Every firm also makes the same risk choice σ'. This greatly simplifies aggregation and renders a simple and tractable model. Notice as well that firm's riskiness can be divided in two components: the leverage she chooses $\boldsymbol{\ell}'(\cdot)$ as well as the risk itself $\boldsymbol{\sigma}'(\cdot)$. Both of these policies will affect default rates, which is the reason why debt prices depend on these policies. In Section 4, I describe how default depends on both leverage $\boldsymbol{\ell}'(\cdot)$ and risk $\boldsymbol{\sigma}'(\cdot)$. Finally, notice that the function $e(l, S)$ is the value of the firm, per unit of capital, after production and debt payments: it accounts for the firms assets (capital) minus firms liabilities, the latter which depend on firms policies.

3.2 Banks

Financial intermediation between firms and households is done by banks, which are run by bankers. The key outcome of the bankers' problem is that their capacity to lend to firms is limited by their net worth. After periods of banks' net worth losses, firms' borrowing will decline which will trigger the firms' riskiness choice.

The sequence of events during a period for a banker is as follows. At the beginning of the period, a banker collects her returns on previous investment (firms' bonds and the risky security), and repays deposits to households. After that, she learns whether she has to exit the economy or not. Exit is exogenous and occurs with probability $1 - \psi$. If the banker exits, she pays her net worth as dividends to the households who are the owners of the bank. If the banker doesn't exit, she can make a new portfolio decision: purchase firms' bonds, holdings of the risky security as well as take new deposits from households. Finally, every period new banks are created to replace the exiting ones. Upon entry, a new bank receive a capital injection equal to a fraction $\bar{\omega}$ of the dividend paid by the exiting bank. Banks' dividends are only allowed upon exit. As I discuss next, banks' face a leverage constraint and will thus find it optimal not to pay positive dividends.[28]

Banks' portfolio decisions are limited by an agency problem as in Gertler and Kiyotaki (2009). This agency problem is meant to capture a limit on banks portfolios in a simple manner. In particular, after making her portfolio decision, a banker can divert a fraction θ of bank's total assets for her own consumption. In order to avoid this, bankers face an incentive constraint so that they decide not to divert assets in equilibrium.

Let $V(N,S)$ be the value to a banker that has net worth N and *did not exit* this period when the aggregate state of the economy is S. Then

$$V(N,S) = \max_{\{b'(l',\sigma'),A',D'\}} \left\{ \mathbb{E}_{S'}\left[m(S,S')\left\{(1-\psi)N' + \psi V(N',S')\right\} |S\right] \right\} \tag{7}$$

subject to

$$\int Q(l',\sigma',S)b'(l',\sigma')dl'd\sigma' + Q^A(S)A' \leq N + Q^D(S)D'$$

$$V(N,S) \leq \theta\left[\int Q(l',\sigma',S)b'(l',\sigma')dl'd\sigma' + Q^A(S)A'\right]$$

$$\begin{aligned} N' &= \left[\xi' + Q^A(S)'\right]A' - D' \\ &+ \int \left[1 - F(\underline{\boldsymbol{a}}(l',S'),\sigma')\right]\left[(c+\lambda) + (1-\lambda)Q(\boldsymbol{\ell}'(l',\sigma',S'),\boldsymbol{\sigma}'(l',\sigma',S'),S')\right]b'(l',\sigma')dl'd\sigma' \end{aligned}$$

The first line in the feasible set is the bank's budget constraint. A bank's investment is divided in purchases of firms' bonds $b(l',\sigma')$ and holdings A' of the risky security. In order to finance her spending, a bank can use her own net worth N or take deposits D' from households, at a price $Q^D(S)$. This portfolio decision is limited by the incentive constraint below: the value to the bank $V(N,S)$ must be larger that the value of diverting a fraction θ of her total assets $\left[\int Q(l',\sigma',S)b'(l',\sigma')dl'd\sigma' + Q^A(S)A'\right]$. The last line in the feasible set is the law of motion for bank's net worth. It includes returns on the risky security ξ' plus next period value of the asset $Q^A(S')$, minus repayment of deposits. Finally, it also accounts for the returns on lending to firms: for a given bond indexed by (l',σ'), a fraction $[1 - F(\underline{\boldsymbol{a}}(l',S'),\sigma')]$ will not default and pay the coupon c plus the maturing fraction λ. Also, the remaining fraction $(1-\lambda)$ has a market value of

[28]Assuming a non-negative dividend constraint would be sufficient. See Gertler and Karadi (2011) for further discussion.

$Q(\boldsymbol{\ell}'(l',\sigma',S'),\boldsymbol{\sigma}'(l',\sigma',S'),S')$, where $\boldsymbol{\ell}'(\cdot)$ and $\boldsymbol{\sigma}'(\cdot)$ are the firms policies next period.

Notice that the incentive constraint links banks' net worth to their capacity to hold assets and thus to lend. This is because the value of the bank $V(N,S)$ is increasing in net wroth N, and thus periods of low net worth imply a tighter incentive constraint. Assuming that banks exit with probability $1-\psi$ is to limit their capacity to accumulate net worth to a point in which the incentive constraint is never binding.[29] This is the modeling structure I borrow from Gertler and Kiyotaki (2009). Second, long-term debt implies that returns on lending partly depends on the firms' actions next period. This is because firms' leverage $\boldsymbol{\ell}'(\cdot)$ and risk $\boldsymbol{\sigma}'(\cdot)$ policies affect the market value of her debt next period. In turn, this dependence amplifies the effect of financial shocks: a decrease in banks' net worth induce decrease in lending, which incentivizes risk taking by firms. This makes the market price of firms lower and, since banks have these bonds in their balance sheet, further depresses banks' net worth

The following proposition characterizes the bank's optimal policies.

Proposition 2 ***(Bank's Optimal Policies)*** *A bank's value function is linear in net worth:*

$$V(N,S) = v(S)N$$

Furthermore, a bank's optimal portfolio decisions are also linear in net worth and given by:

$$b'(l',\sigma',N,S) = \alpha_b(l',\sigma',S)N, \qquad A'(N,S) = \alpha_A(S)N, \quad \text{and} \quad D'(N,S) = \alpha_D(S)N$$

Finally, banks' marginal value of net worth satisfies

$$v(S) = \frac{1}{Q^D(S)}\frac{\mathbb{E}_{S'}\left[m(S,S')\left\{(1-\psi)+\psi v(S')\right\}|S\right]}{1-\mu(S)} \tag{8}$$

where $\mu(S)$ is the Lagrange multiplier associated with the incentive constraint in (7).

Proposition 2 shows that aggregation is very simple at banks level as well. In particular, as a

[29] The incentive constraint implies that banks' marginal value of wealth follows a sub-martingale and thus bankers want to accumulate net worth to a point in which the incentive constraint does not bind in any possible future state. Bankers exogenous exit prevents this. See Chamberlain and Wilson (2000)

fraction of their net worth, every bank makes the same portfolio decision. In turn, this imply that only the total net worth across banks will be relevant and not the particular distribution of net worth. I discuss more of the bank's problem in Section 4.

3.3 Capital Producers

Introducing capital producer is a device to obtain a state dependent price of capital. As I argue below, this serves to amplify the effect of shocks in the economy.

Capital is produced by one period lived firms under perfect competition. They can produce capital using a decreasing returns to scale technology with consumption goods as input. In particular, an amount of investment i delivers $\Phi\left(\frac{i}{K}\right)K$ new units of capital, where K is the total available capital in the economy. The function $\Phi(\cdot)$ is increasing and concave, which captures the difficulty of quickly changing the level of capital installed in the economy.[30] The problem of a capital producer is then as follows

$$\Pi^k(S) = \max_i \left\{ Q^k(S)\Phi\left(\frac{i}{K}\right)K - i \right\} \tag{9}$$

First order conditions for capital producers yields

$$Q^k(S) = \frac{1}{\Phi'\left(\frac{I(S)}{K}\right)} \tag{10}$$

where I already introduced the equilibrium optimal investment level $I(S)$. Since $\Phi(\cdot)$ is concave, equation (10) implies that capital prices will be low in periods of low investment. This is what delivers a state-dependent price of capital in the model.

The importance of capital producers in this model comes from inspecting Proposition 1. During recessions, investment will be low and thus capital prices will be low. Equation (6) then states that the value of firms' installed capital $e(l, S)$ will decline as well. This will increase default and depress

[30]This assumption for capital production has become a standard in the macroeconomic models with financial frictions. See Brunnermeier and Sannikov (2012), Guvenen (2009) and Bocola (2014) among many others. For a survey on modeling assumptions in macroeconomic models with financial frictions see Brunnermeier, Eisenbach, and Sannikov (2013) and Quadrini (2011).

investment even further. Overall, this mechanism amplifies initial disruptions to the economy and is the main reason why I include it in the model.

3.4 Households

The economy has a representative household who makes labor, consumption, and saving decisions every period. Her state is the amount of savings D she deposited at the banks last period. Let $V^H(D,S)$ be the value to a household who has deposits D when the aggregate state of the economy is S. Then,

$$\begin{aligned}
V^H(D,S) &= \max_{C,H,D'} \left\{U(C,H) + \beta\mathbb{E}_{S'}\left[V^H(D',S')|S\right]\right\} \qquad (11)\\
&\text{subject to}\\
C + Q^D(S)D' &\leq D + w(S)H + \Pi^k(S) + \mathcal{D}^B(S) + \mathcal{D}^F(S) - T_H(S)\\
\mathcal{D}^B(S) &= (1-\psi)\bar{N} - \bar{\omega}(1-\psi)\bar{N}\\
\mathcal{D}^F(S) &= \int d(a_i,b_i,k_i,S)\mathbb{I}\left(a_i \geq \underline{\boldsymbol{a}}(l,S)\right) di
\end{aligned}$$

where $\bar{N} = \int N_i di$ is the total net worth held by banks.

The problem of the household in equation (11) is reasonably standard. Household expenditures are consumption C and savings in deposits D'. To afford this, she can use her previous savings D and current labor earnings $w(S)H$, plus dividend paid from all the firms she owns. Dividends of capital producing firms $\Pi^k(S)$ are defined in equation (9). The term $\mathcal{D}^B$ are dividends received from banks net of equity injections to newly created banks. Finally, $\mathcal{D}^F$ is dividend paid by firms: a firm with idiosyncratic state (a,b,k) pays dividends $d(a,b,k,S)$ in aggregate state S conditional on no defaulting, this is when $a \geq \underline{\boldsymbol{a}}(l,S)$. The household also pays lump-sum taxes $T_H(S)$.

3.5 Aggregates and Equilibrium

Let $K'(S) = \int k(a_i,b_i,k_i,S)\mathbb{I}\left(a_i \geq \underline{\boldsymbol{a}}(l,S)\right) di$ be the total amount of capital that non-defaulting firms purchase for the next period. Then, the total investment $I(S)$ that capital producers make must satisfy

$$\Phi\left(\frac{I(S)}{K}\right)K = K'(S) - (1-\delta)K - \Xi F\left(\underline{\boldsymbol{a}}(l,S),\sigma\right)K \tag{12}$$

Equation (12) is a standard law of motion for capital with two additions. On one hand, it incorporates that new investment is produces with a function $\Phi(\cdot)$. On the other hand, equation (12) also accounts for the fraction Ξ of firm's capital that is lost upon default.[31]

For simplicity, I assume that the fraction of defaulting firms' capital that isn't destroyed is seized by the government and rebated to households in lump-sum taxes. This is to keep symmetry between what bond-holders and equity-holders obtain upon a firm's default. The government runs a balanced budget every period which is given as follows

$$0 = T_H(S) + \tau \int \{-\lambda c b_i + a_i k_i \pi(S)\}\,\mathbb{I}(a_i > \underline{\boldsymbol{a}}(l,S))di + (1-\Xi)\,F\left(\underline{\boldsymbol{a}}(l,S),\sigma\right)Q^k(S)K \tag{13}$$

I also assume that the risky security is in fixed supply $\bar{A}$. Then, market clearing for the risky security reads

$$\int A(N_i,S)di = \bar{A} \tag{14}$$

Let $Y(S) = \int y(a_i,b_i,k_i,S)\mathbb{I}\left(a_i \geq \underline{\boldsymbol{a}}(l,S)\right)di$ be total output produced by firms, where $y(a,b,k,s)$ is the output implied by the firm static problem in equation (2). Then, resource feasibility in the economy reads

$$Y(S) + \xi\bar{A} = C(S) + I(S) \tag{15}$$

where $C(S)$ is households consumption.

Finally, the state of the economy is given by the aggregate productivity x, the financial shock ξ, the leverage firms started this period with l, the firms' risk choice carried from last period σ,

[31] Note that, if $\Phi(\cdot)$ is the identity function and there is no default, equation (12) is the standard law of motion in the neoclassical growth model.

total capital in the economy K and total household deposits D: $S = \{x, \xi, l, \sigma, K, D\}$.

A formal equilibrium definition for this economy is as follows.

Definition 1 *A recursive competitive equilibrium for this economy is given by value functions for the firm, bank and household* $\{E, V, V^H\}$*, policies for the firm* $\{d, b', k', \sigma'\}$*, policies for the bank* $\{b'(l', \sigma'), A', D'\}$*, policies for the household* $\{C, H, D'\}$*; such that, given prices* $\{w, Q(l', \sigma'), Q^D, Q^k, Q^A\}$*: (i) firms', banks' and households' policies solve their respective problems and achieve value* E*,* V*, and* V^H *respectively, (ii) the government satisfies his budget constraint, (iii)* ***bonds market*** *clears:* $\int b'(a, b, k, S) = \int b(l', \sigma', N_i, S)di \; \forall l', \sigma'$*, (iv)* ***deposits market*** *clears:* $\int D'(N_i, S)di = D'(D, S)$*, (v)* ***risky security market*** *clears:* $\int A'(N_i, S)di = \bar{A}$*, (vi)* ***goods market*** *clears:* $Y(S) + \xi\bar{A} = C(S) + I(S)$*,*

4 Characterization

In this section I characterize some of the model implications. The aim is to build intuition on the model mechanisms. In particular, I show how a financial shock leads to a decrease in bond prices which induces the firm to deleverage and take on more risk. I start by discussing the asset pricing implications of the bank's problem and then relate this to the firm's leverage and risk decisions.

4.1 Banks Lending and Bond Pricing

I characterize next the outcomes from the banks' problem. The key takeaway of this section is how banks' valuation of wealth affect bond prices.

Proposition 3 ***(Incentive Constraint and Bond Pricing)*** *Let* $v(S)$ *be the marginal value of net worth for a bank as defined in Proposition 2. Then, the incentive constraint in equation (7) reads*

$$\frac{\int Q(l', \sigma', S)b(l', \sigma')dl'd\sigma' + Q^A(S)A'}{N} \leq \frac{v(S)}{\theta} \tag{16}$$

The price of any bond type (l', σ') is given by

$$Q(l', \sigma', S) = \mathbb{E}_{S'} \left[\tilde{m}(S, S') \left[1 - F\left(\underline{\boldsymbol{a}}(l', S'), \sigma' \right) \right] \left[(c + \lambda) + (1 - \lambda) Q(\boldsymbol{\ell}'(l', S'), \boldsymbol{\sigma}'(l', S'), S') \right] | S \right] \tag{17}$$

where $\tilde{m}(S, S')$ is the bank's stochastic discount factor given by

$$\tilde{m}(S, S') = m(S, S') \frac{\{(1 - \psi) + \psi v(S')\}}{\kappa(S) + \theta \mu(S)} \tag{18}$$

where $\kappa(S)$ and $\mu(S)$ are the Lagrange multipliers associated with the banks' budget and incentive constraint in equation (7), respectively.

Equation (16) is a direct implication of the linearity in banks' value function. Notice that the left hand side of equation (16) is bank's leverage (assets over net worth), and thus the incentive constraint yields a leverage constraint for banks. Importantly, this leverage constraint limits banks' lending capacity as a function of their net worth: after periods of capital losses, banks will have to decrease their asset side which imply a decline in lending. This is one of the main outcomes in Gertler and Kiyotaki (2009).

Equations (17) and (18) determine firms' borrowing costs and are central to the model mechanism. Equation (17) is a standard asset pricing equation: the price of a bond $Q(l', \sigma', S)$ is given by the expected pay-off of this asset, weighted by the bank's (buyer) stochastic discount factor $\tilde{m}(S, S')$. In turn, the expected pay-off of the bond accounts for the firm's default probability, as well as the market value of the bond next period in case of repayment. More importantly, the bankers' discounting $\tilde{m}(S, S')$ responds to her current valuation of wealth, which is measured by the multipliers $\kappa(S)$ and $\mu(S)$. After periods of capital losses, banks' marginal value of current wealth increases (a raise in $\kappa(S)$ and $\mu(S)$), which makes the bank discount the future more (a decline in $\tilde{m}(S, S')$) and induce a fall in bond prices. Thus, financial shocks that adversely affect banks net worth, result in a decrease of bond prices. I argue next that this induces firms to issue less debt and increase their risk choice.

Equation (17) is also insightful for understanding the effect of firms' risk choice σ' on bond prices. In the calibrated version of this model, the increase in firms' risk yields an increase in default probabilities which in turn reduces bond prices. This is a cost that the firm faces when increasing her risk choice σ' and a key determinant of the endogenous volatility mechanism developd here. I move next to characterizing the firm's policies.

4.2 Firm's Policies

Firm's default probability is captured by her optimal policy to default, as well as by her leverage and risk decisions. I start by describing her optimal default decision, and the move to her leverage and risk decisions, which is the novelty of this paper.

Firms' Default Decision

As highlighted in Proposition 1, firms' default policy follows a threshold decision. The following proposition characterizes this threshold.

Proposition 4 ***(Default Threshold)*** *A firm defaults if and only if her idiosyncratic productivity is below a threshold* $\underline{\boldsymbol{a}}(l, S)$*. This threshold is determined by*

$$\underline{\boldsymbol{a}}(l, S) = \frac{1}{\pi(S)(1-\tau)} \left[\{(1-\tau)c + \lambda\}\, l - e(l, S)\right] \tag{19}$$

where $e(l, S)$ *is defined in* (6)*. In turn, the probability that the firm defaults next period when the aggregate state of the economy is* S' *is given by* $F\left(\underline{\boldsymbol{a}}(\boldsymbol{\ell}'(l, S), S'), \boldsymbol{\sigma}'(l, S)\right)$*.*

Equation (19) shows an analytical solution for the default policy of the firm and conveys many insight of the model. The default threshold responds to two static components. On one hand, the larger the current profits $\pi(S)$ are, the lower the default threshold. This is intuitive since defaulting firms exit the economy without producing this period. On the other hand, the larger the firm's leverage l is, the larger default threshold. This is also intuitive: the higher the leverage, the more debt payments the firms must met and the larger the incentives to default. The default policy also captures a forward looking component. In particular, the larger the value of capital inside the firm $e(l, S)$, the lower the default threshold. This is the consequence of modeling default decision as a forward looking problem. Finally, notice that the probability of default will also depend on the firm's risk choice σ. Although it doesn't directly affect the default threshold, the risk choice affects the distribution of the idiosyncratic shock $F(a,\sigma)$. As the quantitative evaluation of the model makes clear, increases in σ will sharply increase default rates.

Firms' Leverage Decision

The previous proposition discussed default rates given a leverage choice previously made by the firm. The next proposition characterizes the firms' optimal leverage decision.

Proposition 5 ***(Leverage Decision)*** *Given choices of risk σ' and capital growth ι, the optimal choice of leverage l' satisfies*

$$\begin{aligned} Q(l',\sigma',S) \quad &+ \quad \frac{\partial Q(l',\sigma',S)}{\partial l'}\left[l' - (1-\lambda)\frac{l}{\iota}\right] \\ &= \quad \mathbb{E}_{S'}\left[m(S,S')\left[1 - F\left(\underline{\boldsymbol{a}}(l',S'),\sigma'\right)\right]\left[\{(1-\tau)c + \lambda\} + (1-\lambda)Q\left(\boldsymbol{\ell}'(l',S'),\boldsymbol{\sigma}'(l',S'),S'\right)\right]|S\right] \end{aligned} \tag{20}$$

Equation (20) comes from the firm's first order conditions, and exhibits the marginal costs and benefits of increasing leverage. The right hand side is the expected cost of issuing debt for the firm: with probability $[1 - F(\underline{\boldsymbol{a}}(l',S'),\sigma')]$, the firm will have to repay a fraction λ of her debt plus the coupon c (minus tax benefits), and it will further have issued a liability with market price $Q(\boldsymbol{\ell}'(l',S'),\boldsymbol{\sigma}'(l',S'),S')$ for the remaining fraction of debt $(1-\lambda)$. The left hand side of equation (20) is the marginal benefit of issuing debt: it includes the price per unit of debt $Q(l',\sigma',S)$ plus the change in price for increasing debt $\frac{Q(l',\sigma',S)}{\partial l'}$ times the amount of new debt (tomorrow, per unit of capital) $\left[l' - (1-\lambda)\frac{l}{\iota}\right]$. Notice how the firm internalizes her choice of leverage on the price of debt, which is actually negative and a cost of increasing leverage to the firm. The optimal choice leverage equalizes marginal costs and benefits.

The firm's largest benefit of issuing debt is the price she receives $Q(l',\sigma',S)$. As discussed above, a financial shock that declines banks' net worth, decreases the price of the debt and thus incentivize the firm to deleverage. I show next how this will affect firms' risk choice σ'.

Firms' Risk Choice

Next I discuss the firms' optimal risk choice σ', which is the key novelty in the paper, and how the risk choice relates to the leverage choice l'.

Proposition 6 ***(Risk Choice)*** *Given choices of leverage l' and capital growth ι, the optimal risk choice σ' satisfies*

$$\frac{\partial Q(l',\sigma',S)}{\partial \sigma'}\left[l'-(1-\lambda)\frac{l}{\iota}\right]= \tag{21}$$
$$\frac{\partial}{\partial \sigma'}\mathbb{E}_{S'}\left[m(S,S')\int_{\underline{\boldsymbol{a}}(l',S')}^{\infty}\left\{(1-\tau)\left[\pi(S')a-cl'\right]-\lambda l'+e(l',S')\right\}dF(a,\sigma')|S\right]$$

Equation (21) comes from firms' first order conditions and is the key equation determining the new mechanism in the model. The expectation term on the right hand side of equation (21) is the value of the firm (per unit of capital) next period. Then, the right hand side of equation (21) is the marginal benefit to the firm of increasing σ'. Notice that it includes both the effect on the expected idiosyncratic productivity a, as well as the option value effect on the truncated integral. The left hand side of equation (21) is the marginal cost to the firm of increasing σ'. As argued above, an increase in firms' risk choice σ' decreases bond prices. This is a cost to firm, since she now obtains a smaller inflow for the same promise of future payments. Naturally, the optimal choice of σ' equalizes marginal costs and benefits.

The key insight of equation (21) is that the cost associated with increasing risk σ' crucially depends on the amount of new debt that the firm is issuing. For instance, if the firm is not issuing any new debt ($b'=(1-\lambda)b$), the cost of increasing risk disappears from equation (21).[32] On the contrary, if the firm is funding much of her capital with debt, her leverage is high and thus internalizes more the effect that σ' has on prices. This delivers an new interaction between debt issuance and firms' risk taking: leverage and profits volatility are inversely related in equilibrium.

The logic described above naturally interacts with the financial shocks in the economy. A negative innovation in ξ reduces banks' net worth, which decreases bond prices. Then, firms choose to deleverage which incentivizes to select a riskier technology. As a result, initial negative financial innovations induce an increase in firms' profits volatility.

Long-Term Debt and Corporate Taxes

I conclude this section by explaining the effects of some the assumptions made. The next

[32] To make it clear, notice that $[b'-(1-\lambda)b]=k'\left[l'-(1-\lambda)\frac{l}{\iota}\right]$.

proposition is useful to understand the effects of long-term debt as well as of corporate taxes. In order to ease exposition, I will assume that banks' financial intermediation is unconstrained.[33]

Proposition 7 ***(Effect of Long Term Debt and Corporate Taxes)*** *Assume that banks' incentive constraints are never binding. Then, given choices of risk σ' and capital growth ι, optimal leverage choice for the firm is given by*

$$\boldsymbol{\ell}'(l,S) = \tau c \frac{\mathbb{E}_{S'}\left[m(S,S')\left[1 - F(\underline{\boldsymbol{a}}(\boldsymbol{\ell}'(l,S),S'),\sigma')\right]|S\right]}{-\partial Q(\boldsymbol{\ell}'(l,S),\sigma',S)/\partial l'} + (1-\lambda)\frac{l}{\iota} \tag{22}$$

Equation (22) is analogous to equation (20) where I simply substituted the expression for the bond price (see Proposition 3). There are two interesting implications. First of all, notice that if there is no corporate taxes ($\tau = 0$), equation (22) implies that firm's leverage converges to zero and the firm uses no debt in the long-run.[34] Thus, the only reason why firms use debt in this economy is because of the tax benefit.[35] The second interesting implication of equation (22) is that long term debt generates higher persistence of leverage. If debt has short maturity ($\lambda \approx 1$), debt will respond only to expected default probabilities. In this case, leverage adjusts almost one to one with the arrival of shocks, making it very volatile. In contrast, when debt has long maturity ($\lambda \approx 0$), today's leverage decision has a higher loading on past leverage and becomes less responsive to the arrival of shocks, which makes leverage highly persistent. Furthermore, since leverage becomes less responsive to the state of the economy, the price of debt becomes more volatile, inducing more cyclicality in credit spreads. This is a desirable feature of the data and the main reason why I incorporate long-term debt in the model.[36]

[33]The content of Proposition 7 is unaffected by allowing for constrained financial intermediation, but the algebra gets messier. The consequence of unconstrained financial intermediation is that the stochastic discount factor of the banks is equal to the one of the household $\tilde{m}(S,S') = m(S,S')$. Bond prices are still given as in equation (17).

[34]If $\tau = 0$ and debt is one period ($\lambda = 1$), then leverage is zero in any possible state. If debt has long maturity ($\lambda < 1$), a more precise statement is that, as long as investment is positive ($\iota > 0$), leverage is zero in the non-stochastic steady state. Furthermore, as long as $(1-\lambda) < \iota(S)$ on average (i.e. at the mean of S ergodic distribution), leverage converges to zero with probability one.

[35]This is typically referred in the literature as *trade-off* theory. See Hennessy and Whited (2007) and citations therein.

[36]For a recent discussion of long-term debt in a macro model, see Gomes, Jermann, and Schmid (2013).

5 Quantitative Evaluation

5.1 Calibration

I calibrate most of the parameters in the model to perform the quantitative evaluation. Some of the model parameters are standard and I borrow typical values from the literature. Other parameters are calibrated within the model in order to match certain moments.

A period in the model is a quarter. I use a discount factor of $\beta = 0.99$ which implies an annual risk-free interest rate of 4%. Following Greenwood, Hercowitz, and Huffman (1988), I assume households' utility is given as $U(C, H) = \frac{\left[C - \chi \frac{H^{1+1/\eta}}{1+1/\eta}\right]^{1-\gamma}}{1-\gamma}$. The composite of the utility function eliminates wealth effects from the labor supply. I set the risk-aversion parameter to $\gamma = 1$, and the elasticity of labor to $\eta = 2.5$; both of these parameters values are standard. Finally, I set $\chi = 1.75$ so that households labor supply in steady-state equals one.[37] I set the share of labor in the production function to $1 - \alpha = 0.64$.

I assume a capital production function of the form $\Phi(I/K) = \phi_0 (I/K)^{1-\phi_1} + \phi_2$, as in Jermann (1998). The parameter ϕ_1 determines the elasticity of the price of capital with respect to investment, a key parameter in the model.[38] Following Guvenen (2009), I assume $\phi_1 = 2.5$ which is broadly consistent with the values reported in the empirical literature (for a survey of existing estimates see Christiano and Fisher, 1998). Because this is an important parameter for the model, I provide sensitivity analysis exercises below. I set ϕ_0 so that the price of capital in steady state is one. I assume a depreciation rate of $\delta = 0.025$ and set ϕ_2 to obtain an investment to capital ratio of 1.7% at steady state.

I assume that debt matures at a rate $\lambda = 1/24$, which implies an average maturity of debt of 6 years. This is between the 4 years average maturity reported in Gomes, Jermann, and Schmid (2013) for commercial and industrial loans, and the 8 years average maturity that Gilchrist and Zakrajsek (2012) find in their database on corporate bonds. I set the corporate tax to $\tau = 0.4$ in order to match a steady state ratio of firms' debt to GDP of 3.45, which is the average ratio of total

[37] There is no obvious level for the labor supply, so I take this as a normalization.

[38] From equation (10), and assuming this capital production function, we have that $\frac{\partial \ln Q^k}{\partial \ln I} = \phi_1$, and thus ϕ_1 is the elasticity of capital prices with respect to investment.

credit instruments for non-financial firms over GDP for the period 1951-2013.[39] Gomes, Jermann, and Schmid (2013) also use this value, which is close to the 35% assumed by Jermann and Quadrini (2012).[40] Finally, I set the coupon payment to $c = 0.075$, which delivers a steady state leverage of 45%, in line with non-financial firms in Compustat.[41]

The model's new parameters are those determining the firm's mean-variance tradeoff. I assume that $\mu(\cdot)$ is quadratic in σ and given by $\mu(\sigma) = \mu_a + (\varphi_1 - \varphi_2\sigma)\,\sigma$. I calibrate φ_1 and φ_2 jointly to match two moments of the endogenous (i.e. model implied) process for variance σ. First, I target a steady-state level of $\sigma = 0.4$, which is a moment in the literature to target average default rates.[42] This number is line with the calibration in Carlstrom and Fuerst (1998), and also with the estimations of Christiano, Motto, and Rostagno (2014) who find a posterior value of $\sigma = 0.26$ and an average realization of $\sigma = 0.58$ for the period 1985:1 - 2010:4.[43] The second moment I target is the volatility over time of the cross-sectional dispersion of equity returns. Using a panel of non-financial firms in the CRSP data set, I obtained a volatility of 0.31.[44] The model delivers a close value of 0.34.[45] Finally, the constant μ_a is set so that the expected value of the idiosyncratic productivity a is one in steady-state. The parameters obtained are $\{\mu_a, \varphi_1, \varphi_2\} = \{-0.14, 0.62, 1.17\}$.[46]

The parameter choices for the idiosyncratic distribution imply low default rates.[47] Non-financial corporate default rate is around 1% a year in normal times, while the model only delivers half of that.[48] In order to match the observed default probability, I add an absorbing state to the

[39] Total credits instruments comes from Flow of Funds. See Appendix A for details.

[40] Some argue that the combined (total) corporate tax in the US could be as high as 39%. See http://www.heritage.org/federalbudget/corporate-tax-rate.

[41] Firms' leverage in the model $l = b/k$ corresponds to book leverage. I computed book leverage for non-financial firms in Compustat as total liabilities (LTQ) over total assets (ATQ) using quarterly periodicity data. Non-financial firms are defined as firms with SIC below 6000 or above 6999.

[42] See for instance Carlstrom and Fuerst (1997).

[43] A recent paper by Chugh (2014) highlights that a macro-finance DSGE model as in Christiano, Motto, and Rostagno (2014) require a high average level of volatility in order to obtain significant effects of changes in volatility on other variables. As he claims, these levels are typically higher than what is found in micro panels. My paper is not an exception to this critique.

[44] Appendix C contains the details on how the volatility of the cross-sectional dispersion of equity returns is computed, as well the computations of the model counterpart.

[45] The cross-sectional dispersion of equity returns in the model is a complicated function of many parameters and is thus hard to match an exact number, see Appendix C.

[46] For these parameter values, it holds that expected idiosyncratic productivity is increasing in σ at steady state.

[47] This problem is also found by Covas and Den Haan (2011) in a similar environment.

[48] This is the average for corporate non-financial firms reported by Moody's in their report "Corporate Default and Recovery Rates, 1920-2010". See also Bernanke, Gertler, and Gilchrist (1999).

distribution of the idiosyncratic productivity. In particular, with probability $1 - p$ the firm's idiosyncratic productivity is zero from this period onwards. I set $p = 0.99875$ in order to match an annual default rate of 1%. As discussed above, I assume that a fraction Ξ of the firm's capital is lost upon default. I set $\Xi = 0.12$ as in Bernanke, Gertler, and Gilchrist (1999), which is a conservative measure below the range of 0.20-0.36 that Carlstrom and Fuerst (1997) defended as empirically relevant.[49]

Three parameters relate to banks. I set the probability of surviving to $\psi = 0.975$, which implies an average life span of 10 years. The fraction of assets that bankers can steal is set to $\theta = 0.125$, to obtain an annual credit spread of approximately 0.9% in line with the average credit spread between Baa and Aaa bonds. Finally, I set the fraction of banks' dividend payments that households use to set new banks to $\bar{\omega} = 0.5$, which implies a banks' book leverage of 8, which is in line with the book leverage of commercial banks in Flow of Funds data.[50]

I normalize the supply of the risky asset to one $\bar{A} = 1$. As discussed above, I think of the risky security as the analogous of the real estate sector in the economy. I set the average return of the risky security ξ to $\mathbb{E}[\xi] = 0.4$ so that the contribution of the risky security to total resources in the economy is about 12%, which is the value added of the real estate sector to GDP in NIPA tables.[51]

Finally, I assume that aggregate productivity x and risky security returns ξ follow an AR(1) process in logs with autorregresive coefficients ρ_x and ρ_ξ and variance of innovations σ_x and σ_ξ respectively. I estimate these parameters using Bayesian methods, with prior and posteriors as described in Table 2. I use two observables. First, the series of total factor productivity (TFP) computed by Fernald (2014b) that is adjusted for utilization.[52] I interpret this measure as the "Solow residual" in the model.[53] The second observable is the market value of financial firms,

[49] In my model, Ξ reads as a fraction of capital destruction, while in these other models it is assumed as bankruptcy cost. I prefer a conservative measure of Ξ because it makes the mechanisms in the models less dependent on the actual amount of capital destruction.

[50] Book leverage for commercial banks in Flow of Funds has been decreasing from about 15 in the mid 1980's to 10 by 2010. However, banks in the model account for any type of financial intermediary. Thus, a leverage of 8 for banks is, although lager than the one targeted by Gertler and Kiyotaki (2009), still a conservative target.

[51] Author computations based on NIPA tables. The value added of the real estate sector relative to the rest economy had a trend during the last 50 years in the U.S. A 12% target corresponds to the average for 1995-2005.

[52] The measure of TFP is updated quarterly by John Fernald and can be downloaded here.

[53] The "Solow residual" in the model is $A_t = \frac{Y_t}{K_t^\alpha H_t^{1-\alpha}}$, where Y_t, K_t and H_t stand for output, capital and hours worked respectively.

Table 1: Calibrated Parameters

Parameter	Value	Description	Target / Source
β	0.99	Discount factor	1% annual risk-free rate
γ	1	Risk-aversion	Standard
η	2.5	Labor elasticity	Standard
χ	1.75	Labor disutility	Average hours ≈ 1
α	0.36	Production function	Standard
δ	0.25	Depreciation rate	Standard
ϕ_0	0.002	Capital production function	Price of capital ≈ 1
ϕ_1	2.5	Capital production function	Elasticity of capital price with respect to investment
ϕ_2	0.0036	Capital production function	Investment-to-capital ratio $\approx 1.7\%$
λ	1/24	Debt maturity	Gilchrist and Zakrajsek (2012)
c	0.075	Cupon payment	Firms book leverage $\approx 45\%$
τ	0.4	Corporate Tax	Debt over output ≈ 3.45
μ_a	-0.16	Risk technology	Average productivity ≈ 1
φ_1	0.62	Risk technology	Volatility of equity returns
φ_2	1.17	Risk technology	Volatility of equity returns
ψ	0.975	Bank survival rate	Average life span of 10 years
$\bar{\omega}$	0.5	Banks equity injection rate	Banks leverage ≈ 8
θ	0.125	Banks diverting rate	Credit spread of 0.9%
$\bar{A}$	1	Risky security supply	Normalization
$\mathbb{E}[\xi]$	0.4	Average risky security return	12% of total output
Ξ	0.12	Capital lost upon default	Bernanke, Gertler, and Gilchrist (1999)

which I interpret as the market value of banks in the model. I compute the market value of financial firms by adding the number outstanding shares times the price of these shares for all financial firms in the CRSP data set.[54] Both series are detrended using a linear time trend in logs.

An advantage of my model is that it permits aggregation which allows for standard solution methods. I solve the model using perturbation methods around the non-stochastic steady state of the economy. Around this point, the banks' incentive constraint in (7) is binding.

5.2 Crisis Experiment

In this section I analyze the model's response to a negative financial shock. In particular, I discuss the model's response to an initial innovation in ξ that decreases the market value of banks by 50%,

[54]A financial firm is defined as a firm with an SIC code between 6001 and 6999. See Appendix A for details.

Table 2: Estimated Parameters

Parameter	Prior			Posterior	
	Distribution	Mean	STD	Mode	[5% – 95%]
ρ_x	Beta	0.90	0.1	0.96	[0.93 – 0.98]
σ_x	Inverse Gamma	0.015	0.1	0.008	[0.008 – 0.009]
ρ_ξ	Beta	0.9	0.1	0.97	[0.95 – 0.99]
σ_ξ	Inverse Gamma	0.015	0.1	0.036	[0.027 – 0.043]

which is a number suggestive of the drop in financial firms market value during the last crisis.[55] I show how the financial shock increases firms' riskiness choice which further amplifies the initial financial disruption.

In order to asses the importance of the endogenous volatility mechanism, I also compute the response to a financial shock in a model with constant volatility. The only difference in the model with constant volatility is that firms cannot choose σ. Everything else is identical across both models, including the calibration.[56] I think of the difference between these two model responses as a measure of how much the endogenous volatility amplifies the initial financial shock. Figure 4 plots the model response to a negative innovation in ξ. The red solid line is the response in the benchmark model, and the blue dashed line is the response in a model where volatility is kept constant.

Figure 4 shows the main mechanism of the model. The negative realization in ξ induces a decrease in banks net worth, which in turn reduces banks' capacity to lend through a tightening of the incentive (leverage) constraint. This triggers the new mechanism proposed in the model: since firms' debt issuance decreases, they become less concerned about the price of their debt and find it more attractive to increase the riskiness of their projects. As a consequence, firms increase their σ choice which results in a sharp increase of default rates. The increase in default rates is the key difference between the model with endogenous and fixed volatility. Given that lending to firms becomes riskier, credit spread increase more in the economy with endogenous

[55] As discussed in Section 5.1, the market value of banks in the model is given by their expected present discount value of dividend payments, which is precisely the banks' value function $V(N, S)$.

[56] Both economies also have the same steady state.

volatility.[57] More importantly, firms' borrowing decreases considerably more in the economy with endogenous volatility. This induces a larger decline in investment, which slowly translates into a decline in output and hours. Noticeably, output and hours actually slightly increase on impact, which is the result of firms shifting towards a riskier and also more productive technology. However, almost immediately the economy starts his path towards a deep recession. Overall, the endogenous volatility mechanism more than doubles the effect on output and hours for the same negative innovation in ξ.

While the shock in ξ has an immediate effect on investment, it takes longer to affect output and hours and only does it to a smaller extent. A 20% decrease in investment only achieves a 1.5% decrease in output and a 0.4% decrease in hours worked after several years. As discussed in the introduction, this is a common problem in macro-finance models: a bad financial shock increases the cost of borrowing, which only alter firms decision to invest.[58] Lower investment affects capital accumulation on the margin only, which barely change the total stock of capital. Thus, the consequences of the financial shock for output and hours are small and take long to materialize. Furthermore, because of capital adjustment costs (measured by the parameter ϕ_1), the investment response is sluggish and adds persistence to the model.[59] This small response of output and hours can be overcome by introducing frictions in the labor market so that the cost of hiring reflects changes in borrowing costs.[60] I will introduce some of these frictions next in Section 5.3 when confronting the model with data.

5.3 The 2007-2009 Crisis

This section contains the main exercise performed in the paper. In particular, I test the model's capacity to explain the sequence of events observed during the 2007-2009 crisis. I show that, in

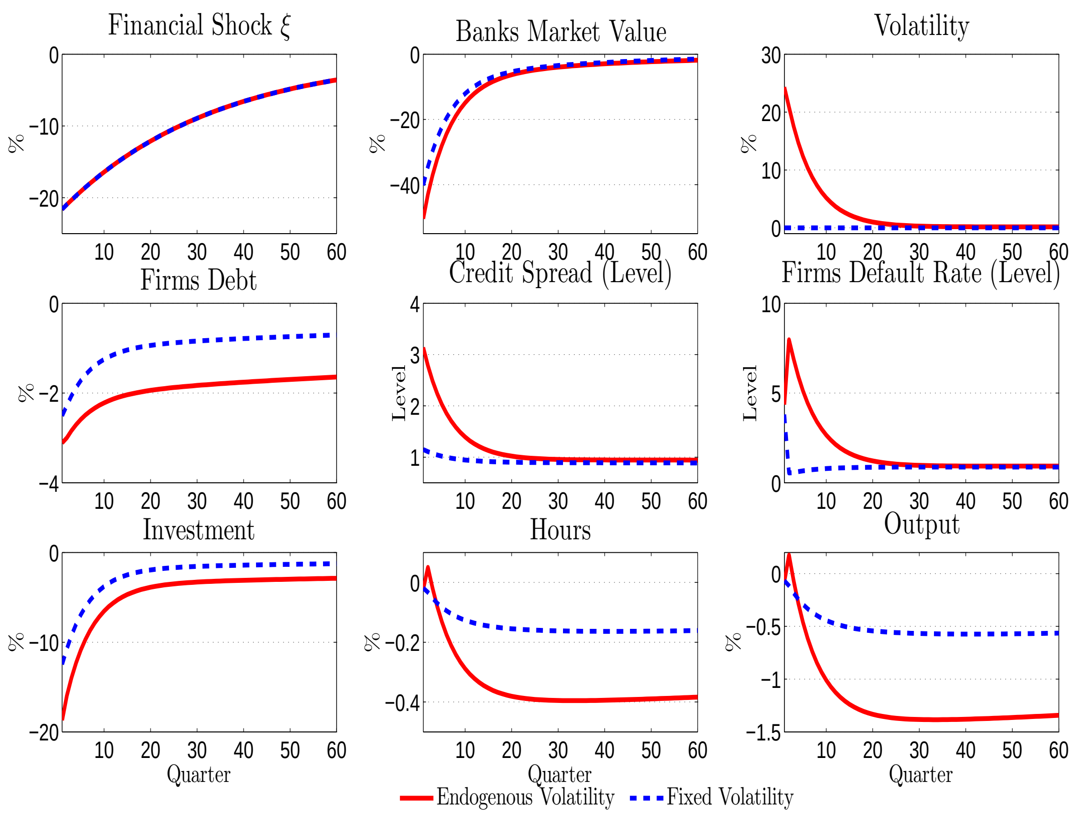
[57]Credit spreads are computed as the difference in the annualized rate of return of the bond issued by the firm with respect to a a bond of the same duration but without risk.

[58]In particular, firm's labor demand and output production decisions are unaffected by the cost of borrowing. This is clearly seen in firm's **static** production decision in equation (2).

[59]Decreasing the value of ϕ_1 makes adjustment in the economy faster, which I show in Figure 10 of Appendix G where I plot the model's impulse response function for the same calibration but decreasing ϕ_1 from 2.5 to 0.25.

[60]Recent papers with these features are Jermann and Quadrini (2012), Bigio (2013), Buera, Fattal-Jaef, and Shin (2013), Garin (2011) and Petrosky-Nadeau (2011) among others. See also Blanco and Navarro (2014) and Monacelli, Quadrini, and Trigari (2011) for papers that provide a microfounded link between financial markets and the labor market by the use of search frictions.

Figure 4: Impulse response function to a negative financial shock ξ.



Notes: *Model response to an innovation in ξ that declines banks' market value by 50%. The red solid line corresponds to the benchmark model and the dashed blue line corresponds to a model with fixed volatility. See main text for details.*

line with the evidence, the model replicates the increase in volatility observed after the the collapse of financial markets. The model can also generate a drop in aggregate variables such as output, investment and hours of similar magnitudes to the ones observed during the crisis. The endogenous volatility mechanism proposed in this paper is key to obtain these large drops.

Additions to the Model: Two Labor Market Frictions

As discussed Section 5.2, financial shocks increase firm's borrowing costs which primarily affect investment but have negligible effects on output and labor. This is clearly an undesirable feature of many macro-finance models. In order to make the model perform better quantitatively, I add two frictions in the labor market. First, I introduce "sticky wages" to the model. In particular, I assume that firms demand labor from a continuum of unions who monopolistically supply a differentiated type of labor but can only reset wages with probability θ_w every period.[61] These unions demand labor from households and produce a unit of the differentiated labor with each unit of households' labor.[62,63] I set $\theta_w = 0.75$, so that unions adjust wages once a year on average.

The second friction I incorporate in the labor market is working capital at the firm level. In particular, firms must pay a fraction θ_L of their wage bill at the beginning of the period, just after making their default decision but before the bond market opens. Thus, firms must finance the fraction θ_L of wage payments by issuing a one period debt. I assume that firms deposit the payment of this short-term debt at the end of the period, and thus cannot default on this debt. However, banks receive the payment next period and then charge an interest rate on the short-term debt. Assuming that firms cannot default on the working capital debt has the purpose of limiting the effect of the endogenous volatility channel on the labor market.[64] I set $\theta_L = 0.67$ that delivers a ratio of short-term liabilities to capital of 0.258, as observed for non-financial firms in Compustat.[65]

[61] Thus, unions subject to a "Calvo type" of friction in adjusting their wages.

[62] There are many ways to introduce wage rigidities in the labor market, all which yield identical equilibrium conditions. See Erceg, Henderson, and Levin (2000) or Gali (2008) chapter 6.

[63] This introduction of "sticky wages" should be understood as a real rigidity, since there are no nominal variables in the model.

[64] In particular, the increase in volatility drives up default rates which would make hiring labor extremely costly. By assuming that firms can commit to repay this debt, the working capital constraint induces an additional cost on labor that only reflects changes in banks' discount factor. For a recent work that analyzes the interaction between default risk and labor cost see Blanco and Navarro (2014) or Gourio (2014).

[65] Compustat defines current liabilities (LCTQ) as "liabilities due within one year, including the current portion of long-term debt". The analogous in the model is $L + (c+\lambda)\left[1 + (1-\lambda) + (1-\lambda)^2 + (1-\lambda)^3\right] B$, where L is firm's working capital borrowing and B is her total debt. As before, I used the total assets series (ATQ) for firm's capital.

Appendix D contains the details of these additions to the model. I also argue below that these additions are important for the quantitative evaluation of the model.

Model Inference During the 2007-2009 Crisis

In order to test the model with data, I perform the following exercise. The model has two aggregate shocks: the financial shock ξ and the productivity shock x. I infer a sequence of shocks so that the model replicates two data observables: the market value of financial firms and firms total factor productivity (TFP), both for the US.[66] Then I compare the model predictions for other (non-targeted) variables with their empirical counterparts during the 2007-2009 crisis. In particular, I test the model on seven other variables: GDP, investment, hours worked, firms borrowing, credit spreads, default rates and the volatility of equity returns.[67]

Figure 5 shows the model predictions for several variables and compares them with data. The black circled lines correspond to the data and red solid lines to the model. For comparison, I also plot in blue dashed lines the model's predictions with fixed volatility. In particular, I filter out the shocks from the model with endogenous volatility and feed these sequences to the model with fixed volatility. All variables are normalized to be zero in the fourth quarter of 2007, which is dated by the NBER as the beginning of the recession. By construction, the model matches the path for financial firms' (banks) market value and productivity.

The model generates a path for the cross-sectional volatility of equity returns that is remarkably in line with the data. At the beginning of 2007, as the market value of financial firms starts to decline, the model predicts an increase in equity returns volatility that reaches a peak of 100% above trend by the end of the recession. A similar path is observed in data, although with a slightly larger increase. The model is equally successful is matching with credit spreads: the model predicts a

[66] I use the same definitions for market value of financial firms and TFP as in Section 5.1.

[67] GDP is computed in 2009 chained dollars. Investment corresponds to real gross private domestic investment, also computed in 2009 chained dollars. Hours worked correspond to hours of all persons in the nonfarm business sector. These three variables are converted to per capita terms by diving by working age population. Annual working age population data is from Organization for Economic Cooperation and Development, and linearly interpolated to quarterly frequency. Firms borrowing corresponds to total credit instruments for non-financial corporate business in the US, which is take from Flow of Funds. Credit spreads corresponds to the measure computed by Gilchrist and Zakrajsek (2012). Default rates are for corporate non-financial firms with speculative grade, the source is Moody's. All data, except credit spreads and default rates, are computed as deviations of a linear time trend in logs. For equity returns volatility I also included a quadratic time trend. Credit spreads and default rates are reported in levels. See Appendix A for more details.

4% increase in spreads by the end of the crisis, slightly below the 5% computed by Gilchrist and Zakrajsek (2012). The model can also replicate the 10% decrease in borrowing during the crisis, although it cannot explain sustained decline in lending after the crisis.

The model also successfully matches the sharp decline in investment, hours worked and GDP during the recession.[68] Between the fourth quarter of 2007 and the second quarter of 2009, the model implies a 6.0% decline in GDP, in line with the 6.2% observed in data; a 34.7% decline for investment, slightly larger than the observed 33.7% decline; and a 8.6% decline in hours worked, close to the observed 8.2% in data.[69] More importantly, the endogenous volatility mechanism is crucial. The same sequence of shock in the model with fixed volatility predicts a GDP decline of 2.3% only, as well as a smaller declines for investment (11.7%) and hours (2.5%). Overall, the model can account for much of the economic performance during the 2007-2009 crisis, and the mechanism proposed in the paper plays a large role in this.

The model's success in replicating the size and, more importantly, the timing of the increase in volatility provides support to the mechanism explored in the paper. In the model, volatility increases slowly and solely as a result of the worsening in financial conditions.[70] This timing of volatility is also observed in data, which hints to the idea of an endogenous component of volatility that is tightly linked to financial conditions.[71] Although the opposite causation cannot be ruled out, a link from financial markets to volatility seems a more reasonable description for the last crisis given that the financial meltdown clearly preceded the rise in volatility.[72]

The transmission from volatility to other variables in the model critically depends on the response of default rates to volatility. As Figure 5 shows, the model predictions for default rates are in line with the pattern observed for the speculative grade bonds in data. Thus, the model transition mechanism does not rely on implausible levels of default rates.

[68] I take GDP in the model to be firms' production only, not accounting for the realizations of the risky security ξ. I do this so that all the model implied decline in GDP is endogenously generated.

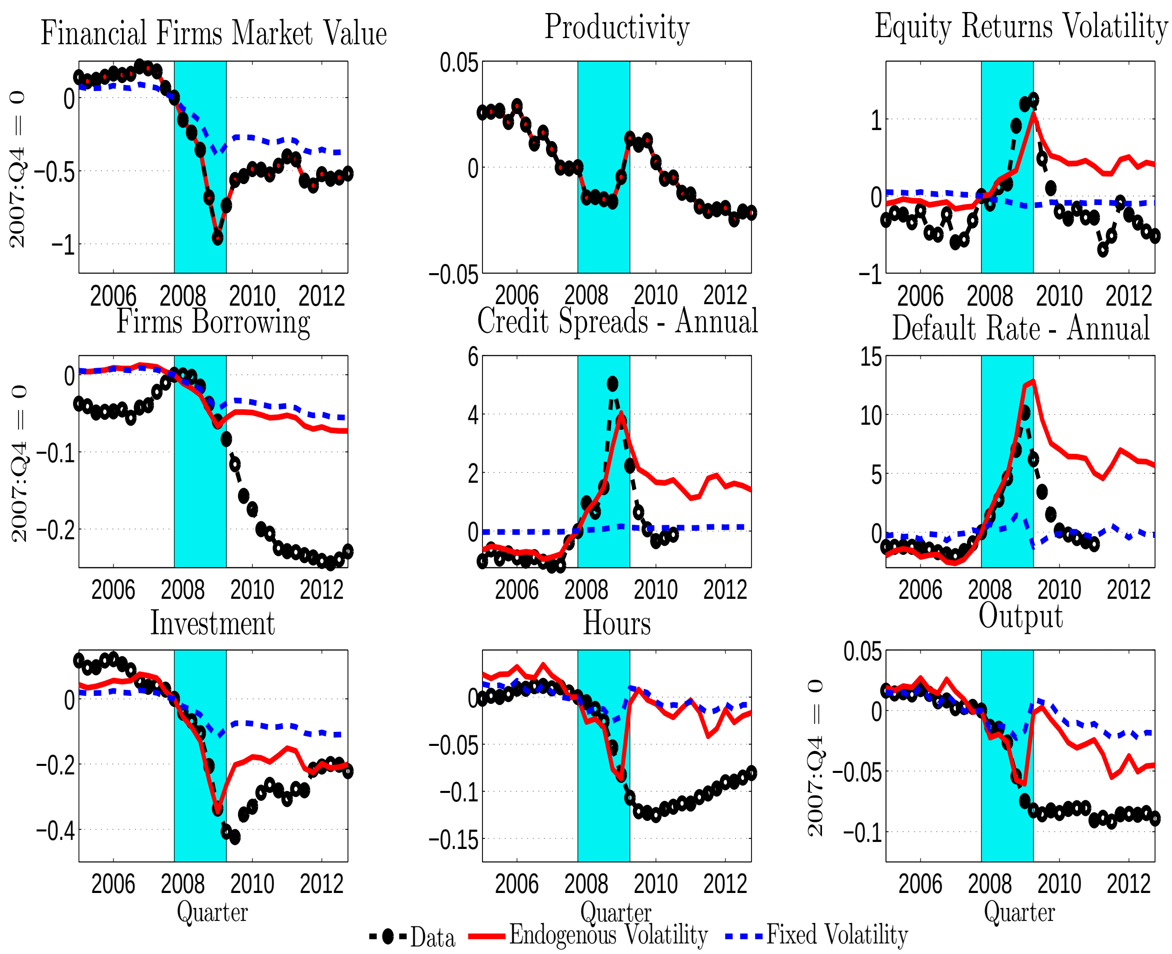
[69] Recall data figures are computed as deviations out of trend.

[70] The results with no innovations in productivity look virtually identical. The plots are available upon request.

[71] See Bachmann, Elstner, and Sims (2013) for very similar findings in a different environments.

[72] Similar conclusions are found in the empirical work of Gilchrist and Zakrajsek (2012). For a different view, see Christiano, Motto, and Rostagno (2014).

Figure 5: 2007-2009 Crisis - Model Inference.



Notes: *Inference for period 1961:1-2012:4. Black circled lines correspond to data and red solid lines corresponds to the benchmark model. The blue dashed lines are the responses in the fixed volatility model to the shocks inferred in the benchmark model. All series normalized to 2007:Q4 = 0. See main text for data construction. Model with* ***flexible wages*** *and* ***working capital****.*

Which Frictions Matter?

Figure 5 shows that the endogenous volatility mechanism significantly amplifies the financial shocks. A natural question is, how did these changes in volatility affect quantities such as output, investment and hours worked? In particular, which frictions are relevant for the transmission mechanism? I answer this next.

As the left panel of Figure 6 shows, The model implies that the inflow of debt decreases considerably more in the economy with endogenous volatility.[73] Although total debt declines by similar amounts in both economies (see Figure 5), the price of the debt decreases significantly more in the economy with endogenous volatility, which is captured by the change in credit spreads in Figure 5. Consequently, the inflow of resources that the firms obtains is considerably smaller in the economy with endogenous volatility, which drives the large difference in investment across both economies.

Because of the increase in default rates, banks' capital losses are larger in the economy with endogenous volatility. This is reflected in higher interest rates and, because of the working capital constraint, in higher labor costs. The right panel in 6 shows the path for the one period risk-free interest rate, which is a cost that firms face in hiring labor. The frictions added to the labor market are then essential for obtaining the decline in hours and consequently on output.

The interaction of sticky wages and the working capital constraint is also key: because wages respond sluggishly, the increase in interest rates affects labor hiring costs more. To see that both of these frictions are needed, Figures 11 and Figure 12 in Appendix G perform the same exercise done in Figure 5 but dropping the sticky wages assumption and the working capital constraint assumption, respectively. As can be seen, eliminating just one of these labor market frictions largely eliminates the drop in the output and hours worked.

A Slow Recovery?

Although the model successfully generates the contraction in economic activity seen during the last recession, it fails to replicate the observed slow recovery. By the end of 2009, the model implies a return to trend for output and hours worked, which is clearly not seen in data. Mechanically, the

[73]The inflow is the change in total debt multiplied by the price of the debt. Let B', B and Q the firms total debt next period, total debt this period and the market price of debt respectively. Then, debt inflow is defined as $Q\left[B' - (1-\lambda)B\right]$. This is the term that comes out in firm's budget constraint in equation (4).

Figure 6: 2007-2009 Crisis - Model Inference.

Notes: *Inference for period 1961:1-2012:4. All series normalized to 2007:Q4 = 0. Red solid lines corresponds to the benchmark model. The blue dashed lines are the responses in the fixed volatility model to the shocks inferred in the benchmark model. See main text for data construction. Model with* ***sticky wages*** *and* ***working capital***

recovery is a result of the increase in productivity as well as of the recovery in the market value of financial firms.

Two comments are important in this regard. First, in this paper I focused on developing a mechanism that could explain the economic downturn during the crisis, and abstracted of features that could induce a slow recovery. Large declines and slow recoveries could naturally be linked, although not the focus of this paper.[74] Second, and more importantly, there is an ongoing debate regarding changes in trend for the U.S. economy which my model cannot account for. Recent discussions on this can be found in Fernald (2014a) and Eggertsson and Mehrotra (2014).[75] I believe that some of the model and data differences after the end of the recession could be the result of these changes in trend.[76] Including some of these low frequency changes in trend is an interesting extension that I left for future research.

6 Evidence

In this section, I provide evidence that supports the main implications of the model. In the model, firms have incentives to decrease their project riskiness in order to reduce the cost of borrowing. Because this incentive is stronger the more the firm borrows, the model predicts that equity returns volatility should be lower the more the firm borrows. I argue next that this implication is supported by the evidence on publicly traded firms in the U.S. I further show that, during the Great Recession, the increase in equity returns volatility was strikingly higher for firms that decreased borrowing the most.

In the model, debt growth and riskiness choices are identical across firms at any given point in time, because policies are linear in the firm's capital.[77] Consequently, I think of the empirical exercises performed in this section as tests of the new mechanism provided in the model, although not of a particular set equations in the model.

[74] See Queralto (2013) for a recent discussion on this.

[75] A very interesting and less formal discussion of this can be found in David Andolfatto's blog here

[76] To show this, Figure 13 in Appendix G replicates the exercise in Figure 5, where output, investment, and hours worked are normalized by labor force instead of working population.

[77] Firms' policies per unit of capital are identical across firms, although they move over the business cycle.

6.1 Evidence: The 2007-2009 Crisis

I start by arguing that during the Great Recession, equity returns volatility experienced a particularly large increase for firms with the largest contraction in borrowing. To do this, I divide non-financial firms in Compustat into quartiles depending on their average debt growth during the last recession (2007:Q4 to 2009:Q2). Then, a firm is in the first quartile if her average debt growth was below the 25% percentile of firms' average debt growth during the recession. Analogous definitions hold *mutatis mutandi* for the other quartiles.[78] As before, non-financial firms are defined as those with SIC codes below 6000 or above 6999. Firms' debt accounts for total liabilities minus deferred tax liabilities.[79]

As before, I compute equity returns volatility as the cross-sectional dispersion of returns: $\hat{\sigma}_t^2 = \frac{1}{N_t}\sum_i (r_{it} - \bar{r}_t)^2$, where N_t is the number of firms in period t, and $\bar{r}_t = \frac{1}{N_t}\sum_i r_{it}$. Analogously, the equity return volatility for firms in quartile n is computed as $\hat{\sigma}_{n,t}^2 = \frac{1}{N_{n,t}}\sum_{i\in n} (r_{it} - \bar{r}_{n,t})^2$, where $N_{n,t}$ is the number of firms in quartile n in period t, and $\bar{r}_{n,t} = \frac{1}{N_{n,t}}\sum_{i\in n} r_{it}$. Firms' returns comes from CRSP; see Appendix A and Appendix C for more details on data sources and computations.

Although volatility increased for all of these groups, there is a clear difference between the first quartile and the others. During the Great Recession, equity returns volatility within firms in the first quartile (lowest debt growth) increased by almost five times more than the increase observed within firms in the fourth quartile (highest debt growth). This can be seen in Figure 7, which plots the volatility of returns for each of the four groups. In particular, the increase in volatility for the first quartile was about 537%, while it was only 107% for the fourth quartile. Overall, the increase in volatility of the first quartile explains about 50% of the total increase in equity returns volatility during the last recession.[80]

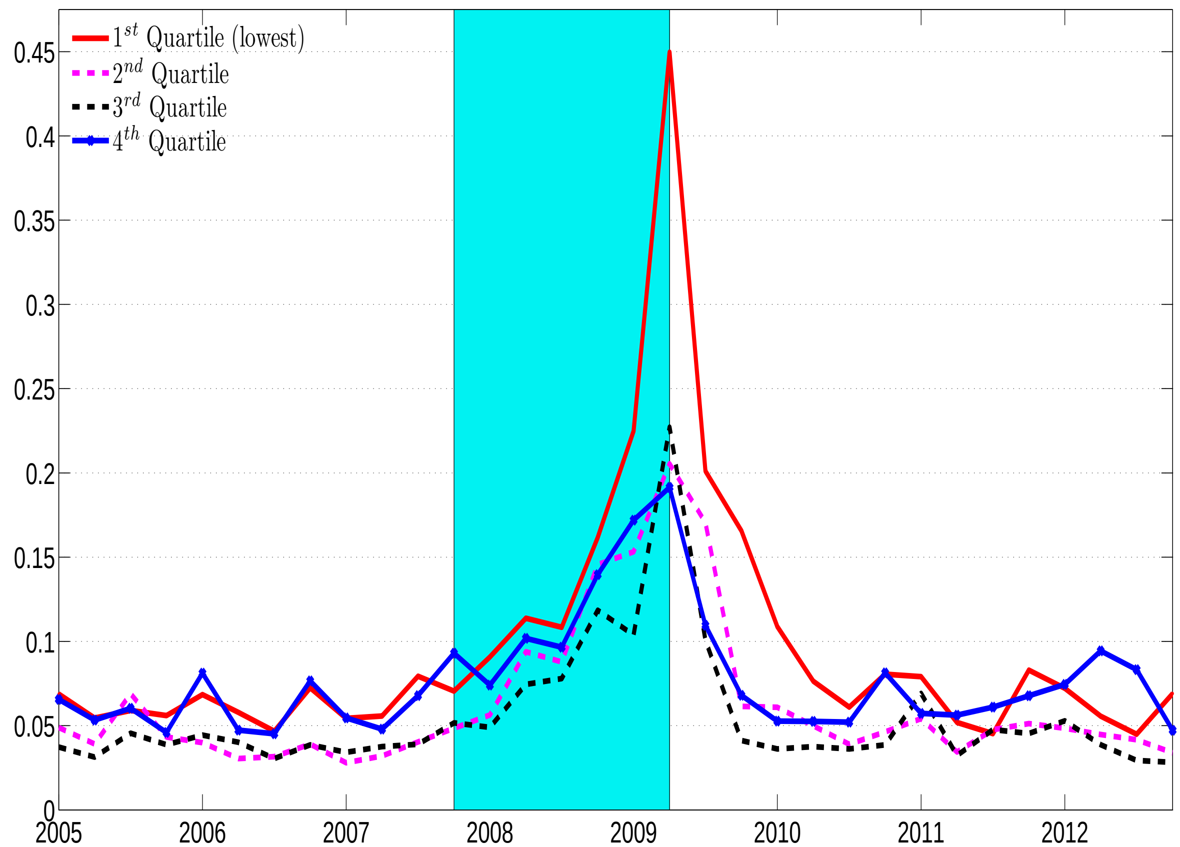
The fact that volatility increased drastically more for firms with the largest decline in borrowing is supporting evidence for the new mechanism proposed in this paper. To complete this discussion, I provide two robustness checks for this finding.

[78] I keep all firms for which I have at least two observations for the period 2007:Q4 to 2009:Q2. Figure 14 in Appendix G plots the thresholds of debt growth that define each one of the quartiles.

[79] Total liabilities in Compustat is the sum of four components: current liabilities, total long-term debt, other liabilities, and deferred taxes and investment tax credit. The measure of debt I use accounts for the first three components only.

[80] This is not immediate from the plot. The data is available upon request.

Figure 7: Equity Returns Volatility within Quartiles of Debt Growth



Notes: *Firms are divided into quartiles according to their average debt growth during 2007:Q4 to 2009:Q2. I keep only non-financial firms. Firms' debt comes from Compustat and is defined as total liabilities minus deferred tax liabilities. Returns come from CRSP. See the main text for more details.*

First, because equity returns volatility increases with a firm's leverage, a reasonable concern is that the different responses in volatility simply reflect differences in firms' leverage. I argue that this is not the case. As can be seen in Figure 15, market leverage for the first quartile had the smallest increase compared to the other groups.[81] Thus, firms' leverage does not seem to be the main driver of equity returns volatility during the last recession. This is in line with the findings of Atkeson, Eisfeldt, and Weill (2014), who also find a negligible effect of leverage in returns volatility.

As a second robustness check, I show that defining quartiles as averages of debt growth during the recession does not drive the results. To do this, every quarter I divide firms into quartiles according to firms' debt growth during that quarter only. Figure 16 plots the equity returns volatility by groups in this case, and shows that the pattern remains virtually identical with volatility increasing considerably more within firms with the lowest debt growth.

The exercise in this section shows that the firms' capacity to borrow during the Great Recession was tightly linked to the volatility of their returns, which is in line with the model predictions. I argue next that this a pattern observed in other time periods as well.

6.2 Evidence: Debt Growth and Volatility of Returns

In this section, I argue that firms increasing their borrowing in a given quarter typically experience a lower volatility of their returns. I find this fact to be robust to several controls. This is in line with the model implications, and extends the findings of the previous section to a longer time period.

For a given firm i, I define the volatility of the firm's returns as $\hat{\sigma}^2_{i,t} = \frac{1}{D_t}\sum_d \left(r_{i,t_d} - \bar{r}_{i,t}\right)^2$, where r_{i,t_d} is the return of holding a share of firm i in day d of quarter t, D_t is the number of days in that quarter, and $\bar{r}_{i,t} = \frac{1}{D_t}\sum_d r_{i,t_d}$. Thus, $\hat{\sigma}^2_{i,t}$ is the (daily) returns' volatility of firm i in quarter t. Using daily data on returns from CRSP, I construct a quarterly panel of firms' volatilities $\left\{\hat{\sigma}^2_{i,t}\right\}$ for the period 1965:Q1 to 2013:Q2.

In order to systematically evaluate the relationship between firms' volatility and their debt

[81] I computed market leverage for the firm as the ratio of book value of debt over the market value of the firm. I would ideally like to use the market vale of firms' debt, but this item is not available on Compustat.

Table 3: Effect of Debt Growth on Returns Volatility $\ln \hat{\sigma}^2_{i,t}$

Debt Growth	-0.07 [$-0.08, -0.07$]	-0.10 [$-0.11, -0.09$]	-0.18 [$-0.19, -0.17$]	-0.18 [$-0.19, -0.17$]
ln Assets		-0.19 [$-0.20, -0.19$]	-0.24 [$-0.24, -0.24$]	-0.20 [$-0.20, -0.20$]
ln Market Leverage			0.24 [$0.24, 0.24$]	0.22 [$0.22, 0.21$]
ln Profits				-0.03 [$-0.03, -0.02$]
R^2	13%	38%	38%	34%
obs	659,333	659,329	635,745	430,100

Notes: *I keep only non-financial firms. Firm's market leverage is the ratio of firm's debt over firm's market value. Firm's debt comes from Compustat and is defined as total liabilities minus deferred tax liabilities. Firm's market value, assets and profits comes from Compustat. Assets corresponds to total assets and profits corresponds to operating income before interest payments and capital depreciation. Returns come from CRSP. See the main text for more details. Number in parenthesis indicate* 95% *confidence intervals.*

growth, I estimate the following equation

$$\ln \hat{\sigma}^2_{i,t+1} = \gamma_t + \chi_i + \beta_{\Delta B} \Delta B_{i,t,t+1} + \alpha_X X_{i,t} + \epsilon_{i,t} \tag{23}$$

where $\Delta B_{i,t,t+1} = \ln B_{i,t+1} - \ln B_{i,t}$ is the growth rate of the firm's debt $B_{i,t}$. Equation (23) includes a time dummy γ_t, a fixed effect per firm χ_i, and a set of controls for firm i in quarter t $X_{i,t}$. Notice that I regress the realized volatility of firm returns in quarter $t+1$ on the firm's debt growth from quarter t to $t+1$. This is consistent with the timing of the firm's decisions in the model.

Firms' incentives to decrease risk during periods of more borrowing should read as a negative estimate of $\beta_{\Delta B}$ in equation (23). Table 3 shows that this is the case even after controlling for several characteristics at the firm level. Then, in any given quarter, I find robust evidence that firms with higher borrowing experience a lower volatility of their returns. Remarkably, after controlling for assets and market leverage, I still find that a firm's debt growth significantly reduces her returns volatility. These findings are in line with the predictions of the model.

7 Conclusions

I argue that during the Great Recession, initial capital losses in the financial sector induced a large increase in volatility which had drastic consequences on economic activity. I develop a model that can account for these features both qualitatively and quantitatively. The key innovation in the model is the risk project selection at the firm level, which induces increases in volatility as the result of financial shocks. Overall, the model can replicate the increase in volatility and the contraction in economic activity during the 2007-2009 crisis, solely as the result of financial disruptions.

This paper is a step towards incorporating foundations of time-varying volatility into dynamic stochastic general equilibrium models. In doing this, I directly assumed a mean-variance technological trade-off for the firm. How to provide better micro-foundations and estimates for this technological trade-off is an important question that I left for future research.[82]

Another dimension that the model misses is the slow recovery after the end the recession. Although I didn't incorporate any features to model to obtain a slow recovery, I believe that the interaction between financial conditions and long-run growth is an interesting question. I also leave this question for future research.

I also provided evidence that supports the main implications of the model. A more detailed analysis on the micro-evidence of firms' risk-taking decisions is next in the research agenda of this project.

Finally, an advantage of a model with endogenous volatility is that is well suited for policy analysis. This model can contribute to understanding how different policies implemented during the crisis affected risk-taking. I think of this as a top future research priority.

[82]A more detailed discussion on this can be found in Appendix E.

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A Data Sources and Definitions

Data for GDP, investment and hours worked are obtained from FRED, at the Federal Reserve Bank of St. Louis. GDP corresponds to 2009 chained dollars. Investment corresponds to real gross private domestic investment, also computed in 2009 chained dollars. Hours worked correspond to hours of all persons in the nonfarm business sector. These three variables are converted to per capita terms by diving by working age population. Annual working age population data is from Organization for Economic Cooperation and Development, and linearly interpolated to quarterly frequency. Firms borrowing corresponds to total credit instruments for non-financial corporate business in the US, which is take from Flow of Funds. It includes corporate bonds, lending from depository institution, credit market instruments, commercial paper, mortgages and other loans and advances. Credit spreads corresponds to the measure computed by Gilchrist and Zakrajsek (2012).

Data for financial and non-financial is from CRSP. A firm is defined as a financial firm if her SIC code is between 6000 and 6999, and is defined as a non-financial firm otherwise. The market value of a firm in a given quarter is computed as number of outstanding shares she has times the value of those shares. The market value of financial firms is the sum of the market value of all financial firms. Returns for firms are also from CRSP data set. Appendix C below explains the computation of the cross-sectional dispersion of firms' returns.

B Proofs and Computations

B.1 Proofs

Next is the proof for firms' profits in Lemma 1.

Proof. (Firm's Profits) First order conditions for labor in (2) yield $n(a,k,S) = axk\left[\frac{1-\alpha}{w(S)}\right]^{\frac{1-\alpha}{\alpha}}$. Then, output of the firm is given by $y(a,k,S) = axk\left[\frac{1-\alpha}{w(S)}\right]^{\frac{1-\alpha}{\alpha}}$. Finally, profits for the firm are $\Pi(a,k,S) = axkw(S)^{-\frac{1-\alpha}{\alpha}}\left\{(1-\alpha)^{\frac{1-\alpha}{\alpha}} - (1-\alpha)^{\frac{1}{\alpha}}\right\}$. This is the expression in equation (3). ■

Next is the proof for firms' optimal policies in Proposition 1.

Proof. (Firm's Optimal Policies) We proceed by guess and verify. Guess that it exist a function $e(l,s)$ such that $E(a,b,k,S) = \mathcal{P} + e(l,S)k$. Then, a firm default if $\mathcal{P} + e(l,s)k < 0$, which only occurs if $a < \frac{1}{(1-\tau)\pi(S)}\left[\{(1-\tau)c+\lambda\}l - e(l,S)\right]$. Let $\underline{\boldsymbol{a}}(l,S) = \frac{1}{(1-\tau)\pi(S)}\left[\{(1-\tau)c+\lambda\}l - e(l,S)\right]$. Using our guess for $E(a,b,k,S)$ and using the budget constraint to substitute dividend payments in (4) we obtain

$$\begin{aligned} e(l,S)k &= \max_{k',b',\sigma'} \Bigg\{ -Q^k(S)\left[k'-(1-\delta)k\right] + Q(l',\sigma',S)\left[b'-(1-\lambda)b\right] \\ &+ \mathbb{E}_{S'}\left[m(S,S')\int_{\underline{\boldsymbol{a}}(l',S')} \left\{(1-\tau)\left[ak'\pi(S')-cb'\right]-\lambda b'+e(l',S')k'\right\} dF(a,\sigma')\right]\Bigg\} \end{aligned}$$

Then, dividing both sides by current capital k we have

$$\begin{aligned} e(l,S) &= \max_{\iota,l',\sigma'} \Bigg\{ -Q^k(S)\left[\iota-(1-\delta)\right] + Q(l',\sigma',S)\left[l'\iota-(1-\lambda)l\right] \\ &+ \iota\mathbb{E}_{S'}\left[m(S,S')\int_{\underline{\boldsymbol{a}}(l',S')} \left\{(1-\tau)\left[a\pi(S')-cl'\right]-\lambda l'+e(l',S')\right\} dF(a,\sigma')\right]\Bigg\} \end{aligned} \tag{24}$$

where $\iota = k'/k$ and $l' = b'/k'$.

Equation (24) recursively defines $e(l,S)$ which in turn verifies our initial guess of the value function. Also, it shows that policies for capital growth $\iota = k'/k$ and next period leverage $l' = b'/k'$ are independent of any idiosyncratic firm's variable. This verifies that firm's policy functions are linear in capital.

It is straightforward to show that, by taking first order conditions with respect to ι in (24) and plugging the expression back in (24), we obtain equation (6). ■

Next is the proof for banks' optimal policies in Proposition 2.

Proof. (Bank's Optimal Policies) We proceed by guess and verify. Guess that it exist a function $v(S)$ such that $V(N,S) = v(S)N$. Also, without loss of generality, let $\alpha_b(l',\sigma') = \frac{b'(l',\sigma')}{N}$ for an arbitrary policy $b'(l',\sigma')$. Analogously define $\alpha_A = \frac{A'}{N}$ and $\alpha_D = \frac{D'}{N}$. Then, using the law of motion for N' in (7), we have that

$$\begin{aligned} \frac{N'}{N} &= \left[\xi' + Q^A(S)'\right]\alpha_A - \alpha_D' \\ &+ \int \left[1-F(\underline{\boldsymbol{a}}(l',S'),\sigma')\right]\left[(c+\lambda)+(1-\lambda)Q(\boldsymbol{\ell}'(l',\sigma',S'),\boldsymbol{\sigma}'(l',\sigma',S'),S')\right]\alpha_b(l',\sigma')dl'd\sigma' \end{aligned} \tag{25}$$

Using equation (25) with our guess $V(N,S)=v(S)N$ in the objective function of (7) delivers

$$
\begin{aligned}
v(S) = \max_{\alpha_b(l',\sigma'),\alpha_A,\alpha_D} & \Bigg\{ \mathbb{E}_{S'} \Bigg[m(S,S')\left\{(1-\psi)+\psi v(S')\right\} \times \dots \\
& \times \Bigg\{ \left[\xi' + Q^A(S)'\right]\alpha_A - \alpha_D' + \dots \\
& + \int \left[1 - F(\underline{\boldsymbol{a}}(l',S'),\sigma')\right]\left[(c+\lambda)+(1-\lambda)Q(\boldsymbol{\ell}'(l',\sigma',S'),\boldsymbol{\sigma}'(l',\sigma',S'),S')\right]\alpha_b(l',\sigma')dl'd\sigma' \Bigg\} | S \Bigg] \Bigg\}
\end{aligned} \tag{26}
$$

subject to

$$
\begin{aligned}
& \int Q(l',\sigma',S)\alpha_b(l',\sigma')dl'd\sigma' + Q^A(S)\alpha_A \leq 1 + Q^D(S)\alpha_D \\
& v(S) \leq \theta\left[\int Q(l',\sigma',S)\alpha_b(l',\sigma')dl'd\sigma' + Q^A(S)\alpha_A\right]
\end{aligned}
$$

The recursion in equation (26) does not depend on net worth N. This verifies our initial guess for the value function and shows that policies $\alpha_b(l',\sigma')$, α_A and α_D are independent of net worth. ■

Next is the proof for Proposition 3 regarding bank's incentive constraint and the bond pricing equation.

Proof. (Incentive Constraint and Bond Pricing) From Proposition 2 we know that bank's value function is given as $V(N,S)=v(S)N$. Introducing this expression in the incentive constraint in equation (7) delivers the leverage constraint in equation (16). The bond pricing formula in equation comes from first order conditions in the normalized bank's problem in equation (26). ■

Next is the proof for firms' default policy in Proposition 4.

Proof. (Default Threshold) From Proposition 1 we have that $E(a,b,k,S)=(1-\tau)\left[ak\pi(S)-cb\right]-\lambda b+e(l,S)k$, where I already used the definition of $\mathcal{P}$ in (4). Since the outside value to a firm is zero, a firm find optimal to default if and only if $E(a,b,k,S)<0$. Simple algebra yields that $E(a,b,k,S)<0$ if and only if $a<\frac{1}{(1-\tau)\pi(S)}\left[\{(1-\tau)c+\lambda\}l-e(l,S)\right]$. This is the default threshold in equation (19). ■

Proofs of Proposition 5 and Proposition 6 regarding firm's leverage and risk policies.

Proof. (Firm's Leverage and Risk Decisions) Equations (20) and (21) comes directly from first order conditions in firm's normalized problem in equation (24). ■

Proof. First order conditions in households problem, yield the following expression for the price of deposits

$$
Q^D(S) = \mathbb{E}_{S'}\left[m(S,S')|S\right] \tag{27}
$$

If bank's incentive constraint is never binding, then the Lagrange multiplier is always zero: $\mu(S)=0 \ \ \forall S$.

Introducing equation (27) in the expression for $v(S)$ in equation (8), and assuming $\mu(S) = 0$, reads

$$v(S) = \frac{\mathbb{E}_{S'}\left[m(S,S')\left\{(1-\psi) + \psi v(S')\right\}\right]}{\mathbb{E}_{S'}\left[m(S,S')|S\right]} \tag{28}$$

It easy to see that $v(S) = 1 \ \ \forall S$ satisfies equation (28). Then, bank's discount factor in equation (18) implies $\tilde{m}(S,S') = m(S,S') \ \ \forall S,S'$. In turn, the bond price in equation (17) now reads

$$Q(l',\sigma',S) = \mathbb{E}_{S'}\left[m(S,S')\left[1 - F\left(\underline{\boldsymbol{a}}(l',S'),\sigma'\right)\right]\left[(c+\lambda) + (1-\lambda)Q(\boldsymbol{\ell}'(l',S'),\boldsymbol{\sigma}'(l',S'),S')\right]|S\right] \tag{29}$$

Introducing equation (29) in equation (20) and doing some simple algebra yields equation (22). ■

B.2 Computations

The model is solved by using perturbation methods. This accounts for linearinzing the system of equations that characterize the model equilibrium around a particular point, in my case the non-stochastic steady state.

Part of model equations include the derivative of the price function with respect to leverage. Because debt has long maturity, this could be a problem for the linearization since it accounts for computing derivatives of policies functions. I show next an envelope argument that shows that this is not the case in my model.

From equation (17), we can compute the derivative of the price with respect to leverage as

$$\begin{aligned}
\frac{\partial Q(l',\sigma',S)}{\partial l'} &= -\mathbb{E}_{S'}\left[\tilde{m}(S,S')f(\underline{\boldsymbol{a}}(l',S'),\sigma')\frac{\partial \underline{\boldsymbol{a}}\boldsymbol{(l',S')}}{\partial l'}\left[(c+\lambda) + (1-\lambda)Q\left(\boldsymbol{\ell}'(l',S'),\boldsymbol{\sigma}'(l',\sigma',S),S'\right)\right]\right] \\
&+ \mathbb{E}_{S'}\left[\tilde{m}(S,S')\left[1 - F(\underline{\boldsymbol{a}}(l',S'),\sigma')\right](1-\lambda)\left\{\frac{\partial Q\left(\boldsymbol{\ell}'(l',S'),\boldsymbol{\sigma}'(l',S),S'\right)}{\partial l'}\frac{\partial \boldsymbol{\ell}'(l',S')}{\partial l'}\right.\right. \\
&+ \left.\left.\frac{\partial Q\left(\boldsymbol{\ell}'(l',S'),\boldsymbol{\sigma}'(l',S),S'\right)}{\partial \sigma'}\frac{\partial \boldsymbol{\sigma}'(l',S')}{\partial l'}\right\}|S\right]
\end{aligned} \tag{30}$$

Equation (30) shows the challenge, in principle the slopes of policies $\frac{\partial \boldsymbol{\ell}'(l',S')}{\partial l'}$ and $\frac{\partial \boldsymbol{\sigma}'(l',S')}{\partial l'}$ should be computed. However, applying Benveniste and Scheinkman in equation (24) we have that

$$\frac{\partial e(l,S)}{\partial l} = -(1-\lambda)Q\left(\boldsymbol{\ell}'(l',S'),\boldsymbol{\sigma}'(l',S'),S\right) \tag{31}$$

Similarly, taking derivatives with respect to l in equation (6) we have

$$\begin{aligned}\frac{\partial e(l,S)}{\partial l} &= -(1-\lambda)Q\left(\boldsymbol{\ell}'(l,S),\boldsymbol{\sigma}'(l,S),S\right) \\ &- (1-\lambda)\frac{\partial Q\left(\boldsymbol{\ell}'(l,S),\boldsymbol{\sigma}'(l,S),S\right)}{\partial l}\frac{\partial \boldsymbol{\ell}'(l,S)}{\partial l}l \\ &- (1-\lambda)\frac{\partial Q\left(\boldsymbol{\ell}'(l,S),\boldsymbol{\sigma}'(l,S),S\right)}{\partial \sigma}\frac{\partial \boldsymbol{\sigma}'(l,S)}{\partial l}l\end{aligned} \tag{32}$$

For states where $l \neq 0$, equations (31) and (32) together imply

$$\begin{aligned}0 &= \frac{\partial Q\left(\boldsymbol{\ell}'(l,S),\boldsymbol{\sigma}'(l,S),S\right)}{\partial l}\frac{\partial \boldsymbol{\ell}'(l,S)}{\partial l} \\ &+ \frac{\partial Q\left(\boldsymbol{\ell}'(l,S),\boldsymbol{\sigma}'(l,S),S\right)}{\partial \sigma}\frac{\partial \boldsymbol{\sigma}'(l,S)}{\partial l}\end{aligned} \tag{33}$$

Given that equation (33) holds for all S, we can introduce it in (30) to eliminate the computation of policies derivatives. Finally, the price derivative reads as follows

$$\frac{\partial Q(l',\sigma',S)}{\partial l'} = -\mathbb{E}_{S'}\left[\tilde{m}(S,S')f(\underline{\boldsymbol{a}}(l',S'),\sigma')\frac{\partial \underline{\boldsymbol{a}}\boldsymbol{(l',S')}}{\partial l'}\left[(c+\lambda)+(1-\lambda)Q\left(\boldsymbol{\ell}'(l',S'),\boldsymbol{\sigma}'(l',\sigma',S),S'\right)\right]|S\right]$$

No policy slope computation is needed.

C Equity Returns Volatility: Model and Data

In this section I briefly describe how to compute the volatility of equity returns in the model and how I compare it with data. In order to make notation simple, I move to time domain notation (indexed by t) instead of state-space notation (indexed by S) as in te main text.

C.1 Data

I use returns data for non-financial firm from the CRSP data set. A firm is considered to be a non-financial firm if her SIC code is below 6000 or above 6999. Let r_{it} be the return of holding a stock of firm i during quarter t (including dividend payments). Then, for every quarter t I compute the standard deviation of equity returns as

$$\hat{\sigma}_t^E = \sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}(r_{it}-\bar{r}_t)^2} \tag{34}$$

where $\bar{r}_t = \frac{1}{N_t}\sum_{i=1}^{N_t} r_{it}$ and N_t is the number of firms in quarter t.

C.2 Model

Let $r_{i,t-1,t}$ be the return of holding a unit of stock of firm i from period $t-1$ to t. Then,

$$r_{i,t-1,t} = \frac{E_{i,t}}{E_{i,t-1} - d_{i,t-1}} \tag{35}$$

where $E_{i,t}$ is the, *ex-ante dividend*, equity value of the firm as defined in (4). The denominator in (35) adjust for last period dividend payment $d_{i,t-1}$.

From equation (4), note that

$$E_{i,t-1} - d_{i,t-1} = \mathbb{E}_{t-1,a}\left[m_{t,t+1}\max\left\{0, E_{i,t}\right\}\right] \tag{36}$$

where the expectation in (36) is computed over possible aggregate and idiosyncratic realizations next period. From Proposition 1, we know that $E_{i,t} = (1-\tau)\left[a_{it}k_{it}\pi_t - cb_{it}\right] - \lambda b_{it} + e_t k_{it}$, where $k_{i,t}$, $b_{i,t}$ and $a_{i,t}$ are the firm's capital, total debt and idiosyncratic productivity respectively. Since every firm has the same book leverage $l_t = b_{it}/k_{it}$, we have that

$$\frac{E_{it}}{k_{it}} = (1-\tau)\left[a_{i,t}\pi_t - cl_t\right] - \lambda l_t + e_t \tag{37}$$

Introducing (36) and (37) in (35) we have

$$r_{i,t-1,t} = \frac{(1-\tau)\left[a_{it}\pi_t - cl_t\right] - \lambda l_t + e_t}{\mathbb{E}_{t-1,a}\left[m_{t,t+1}\max\left\{0, (1-\tau)\left[a_{it}\pi_t - cl_t\right] - \lambda l_t + e_t\right\}\right]} \tag{38}$$

Notice that, since in equilibrium every firm chooses the same distribution for the idiosyncratic shock, the denominator in (38) is identical across firms. Then, the variance of equity returns is given by

$$Var(r_{i,t-1,t}) = \Upsilon_t^2 Var(a_{it}) \tag{39}$$

where $\Upsilon_t = \frac{(1-\tau)\pi_t}{\mathbb{E}_{t-1}[m_{t,t+1}\max\{0,(1-\tau)[a_{it}\pi_t - cl_t] - \lambda l_t + e_t\}]}$.

D Model with Sticky Wages and Working Capital

In Section 5.3 I introduced two frictions on the labor market: namely, sticky wages and a working capital constraint. In this section I explain how the model changes once sticky wages and working capital are added to the model.

D.1 Firm

There are two changes in firms' problem. First, firms demand a continuum of different type of labor from unions. Second, firms must borrow a short-term loan in order to finance a fraction θ_L of their wage bill. Since firms cannot default on this short-term debt, these two features only affect the production stage of the firm. In particular, firms production problem is now as follows

$$\Pi(a,k,S) = \max_{\{n_i\}} \left\{ (axk)^{\alpha} \left(\int n_i^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}(1-\alpha)} - (1-\theta_L) \int w_i(S) n_i di - \theta_L R(S) \int w_i(S) n_i di \right\} \quad (40)$$

Equation (40) contains the two additions to the model. Firms demand a continuum of differentiated type of labor $\{n_i\}$, each one with a wage $w_i(S)$. The firm then combines differentiated labor with a Dixit-Stiglitz aggregator with elasticity of substitution ϵ_w. Also, firms must borrow a fraction θ_L of their wage bill $\int w_i(S) n_i di$ at the beginning of the period. The interest rate associated with this loan is $R(S)$.

Taking first order conditions in equation (40) we can obtain firm demand of labor type i as

$$n_i = \left(\frac{w_i(S)}{w(S)} \right)^{-\epsilon_w} n \quad (41)$$

where $n = \left[\int n_j^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$ and $w(S) = \left[\int w_i(S)^{1-\epsilon_w} di \right]^{\frac{1}{1-\epsilon_w}}$. Equation (41) is the standard demand equation obtained as a result of the Dixit-Stiglitz aggregator.

Introducing equation (41) in (40) and doing some painful but straightforward algebra, it can be shown that profits are still linear in capital and given as

$$\Pi(a,k,S) = ak\pi(S) \quad (42)$$

where $\pi(S) = x \left[w(S) \left[1 + \theta_L \left(R(S) - 1 \right) \right] \right]^{-\frac{1-\alpha}{\alpha}} \Theta$ and $\Theta = (1-\alpha)^{\frac{1-\alpha}{\alpha}} - (1-\alpha)^{\frac{1}{\alpha}}$.

Profits per unit of capital in equation (42) incorporate the effect of the cost of borrowing $R(S)$ as an additional cost to the a wage $w(S)$, which is in turn a geometric average of the different union wages $\{w_i(S)\}$.

However, the intertemporal firm problem in equation (4) is, *mutatis mutandis* substituting the expression for profits, unaffected by the additions included in the labor market. Then, the results of Proposition 1 still hold.

D.2 Unions

I explain next the problem for the unions. For notational convenience, I move to time domain notation (indexed by t) instead of state-space notation (indexed by S) as in te main text.

Labor supply is done by monopolistic unions that "re-package" the labor they demand from households. Let w_t^H be the market wage at which they hire labor from households, and w the price at which they sell their differentiated labor. Since unions monopolisitically supply a differentiated type of labor, they can only choose the price w. However, they are subject to a subject to a Calvo-type friction and can only reset wages every period with probability θ_w. Then, unions solve the following problem.

$$\begin{aligned} \Pi^L &= \max_{w_j} \mathbb{E}_t \sum_{\tau=0}^{\infty} \theta_w^{\tau} m_{t,t+\tau} \left[w_j - w_{t+\tau}^H \right] n_{j,t+\tau} \\ \text{subject to} \\ n_{j,t+\tau} &= \left(\frac{w_j}{w_{t+\tau}} \right)^{-\epsilon_w} N_{t+\tau} \end{aligned} \tag{43}$$

where the constraint in equation (43) is the firms (aggregate) labor demand in equation (41).

First order condition in (43) deliver

$$0 = \mathbb{E}_t \sum_{\tau=0}^{\infty} \theta_w^{\tau} m_{t,t+\tau} \left(\frac{w_j}{w_{t+\tau}} \right)^{-\epsilon_w} N_{t+\tau} \left[w_j - \mathcal{M}_w w_{t+\tau}^H \right] \tag{44}$$

where $\mathcal{M}_w = \frac{\epsilon_w}{\epsilon_w - 1}$.

Let w_t^* be the solution to (44). The average wage in the economy w_t then is

$$w_t = \left[\theta_w w_{t-1}^{1-\epsilon_w} + (1-\theta_w) w_t^{*1-\epsilon_w} \right]^{\frac{1}{1-\epsilon_w}} \tag{45}$$

D.3 Banks

Banks' problem now includes lending the working capital resources in short-term debt to firms. Banks' problem now read as follows

$$V(N,S) = \max_{\{b'(l',\sigma'),L',A',D'\}} \left\{ \mathbb{E}_{S'}\left[m(S,S')\left\{ (1-\psi)N' + \psi V(N',S') \right\} |S \right] \right\} \tag{46}$$

subject to

$$\int Q(l',\sigma',S)b'(l',\sigma')dl'd\sigma' + Q^A(S)A' + L' \leq N + Q^D(S)D'$$

$$V(N,S) \leq \theta \left[\int Q(l',\sigma',S)b'(l',\sigma')dl'd\sigma' + L' + Q^A(S)A' \right]$$

$$\begin{aligned} N' &= \left[\xi' + Q^A(S)' \right] A' - D' + R(S)L' \\ &+ \int \left[1 - F(\underline{\boldsymbol{a}}(l',S'),\sigma') \right] \left[(c+\lambda) + (1-\lambda)Q(\boldsymbol{\ell}'(l',\sigma',S'),\boldsymbol{\sigma}'(l',\sigma',S'),S') \right] b'(l',\sigma')dl'd\sigma' \end{aligned}$$

where L' is the short-period debt lent to firms and $R(S)$ is the return charged on these loans.

It is straightforward to show that Proposition 2 still holds with the addition of short-term loans. In turn, the interest rate on these loans satisfies

$$1 = R(S)\mathbb{E}_{S'}\left[\tilde{m}(S,S')|S \right] \tag{47}$$

where $\tilde{m}(S,S')$ is the bank's stochastic discount factor.

E Mean-Variance Trade-Offs: Examples

In this section I develop two simple example that provide micro-foundations for the mean-variance trade-off assume in the model.

E.1 A Hedging Policy

Assume of a firm who can produce output with two inputs: capital k and an imported input g.[83] Every period, capital is given by decisions made in the past by the firm, but she can decide how many imported inputs g to buy. The input has a price p, which is determined in international markets and can be described by a uniform distribution $p \sim U[p_L, p_H]$.[84] Let μ_p be the expected value of p: $\mu_p = \frac{p_L + p_H}{2}$.

[83] It is straightforward to add labor in the example without changing any of the implications. I don't include labor just to keep the algebra simple.

[84] A typical example for this imported input could be oil. Assuming that is uniformly distributed just makes algebra easier.

Every period, before learning the realization of p, the firm can hedge against fluctuations in p. In particular, the firm can buy a future on g that guarantees the purchase a fraction h of her inputs g at a *fixed price* $\bar{p}$.[85] I further assume that issuers of these features require a premium, and thus $\bar{p} > \mu_p$.

Then, after the realization of p, and given an amount of capital k and a hedging decision h, firm's problem is as follows

$$\Pi(k, h) = \max_{g} \left\{ (xk)^{\alpha} g^{1-\alpha} - [(1-h)p + h\bar{p}] \, g \right\} \tag{48}$$

As in the model above, constant returns to scale in equation (48) implies profits linear in capital as follows

$$\Pi(k, h) = xk\pi(h, p)$$

where

$$\pi(h, p) = [(1-h)p + h\bar{p}]^{-\frac{1-\alpha}{\alpha}} \, \Theta \tag{49}$$

where $\Theta = (1-\alpha)^{\frac{1-\alpha}{\alpha}} - (1-\alpha)^{\frac{1}{\alpha}}$.

Since $\bar{p} > \mu_p$, equation (49) implies that the expected value of $\pi(h, p)$ is decreasing in h: $\frac{\partial \mathbb{E}_p[\pi(h,p)]}{\partial h} < 0$. Similarly, the variance of $\pi(h, p)$ is clearly decreasing in h being zero for $h = 1$: $\frac{\partial Var_p[\pi(h,p)]}{\partial h} < 0$. Then, the firm face a mean-variance trade-off: she can reduce the volatility of her profits to the cost of reducing her expected returns.

Figure 8 shows the relation between expected return and variance that the firm can achieve. Figure 9 shows the particular level of hedging needed for each point of the mean-variance trade-off.

F Endogenous Volatility: A One Period Example

There are two key assumptions that drive the risk choice σ' made by the firm. On one hand, firms have incentives to increase σ' since they move towards a technology with a higher expected pay-off. On the other hand, given that bond prices decreases with σ', firms have incentives to lower their σ' choice. In this appendix, I develop a simpler two period model to argue that both of these assumptions are necessary to obtain reasonable implications for σ'.

[85] This could be done, for instance, by buying a future on g.

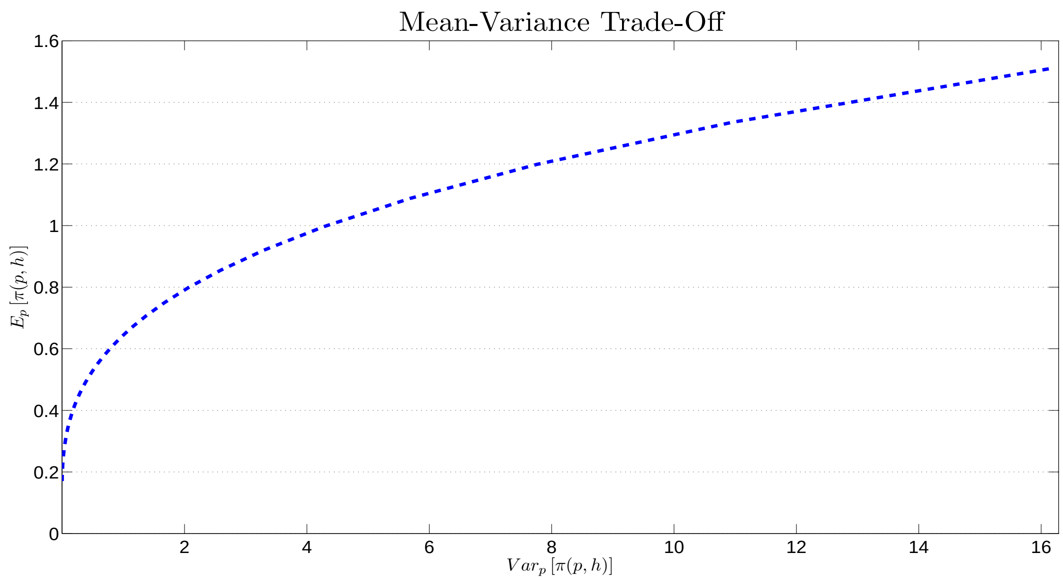


Figure 8: Endogenous Mean-Variance Trade-Off for a Hedging Policy..

Notes: *Parameters used* $p_L = 0.05$, $p_H = 4$, $\bar{p} = 2.43$ *and* $\alpha = 0.33$. *See Appendix E.1 for details.*

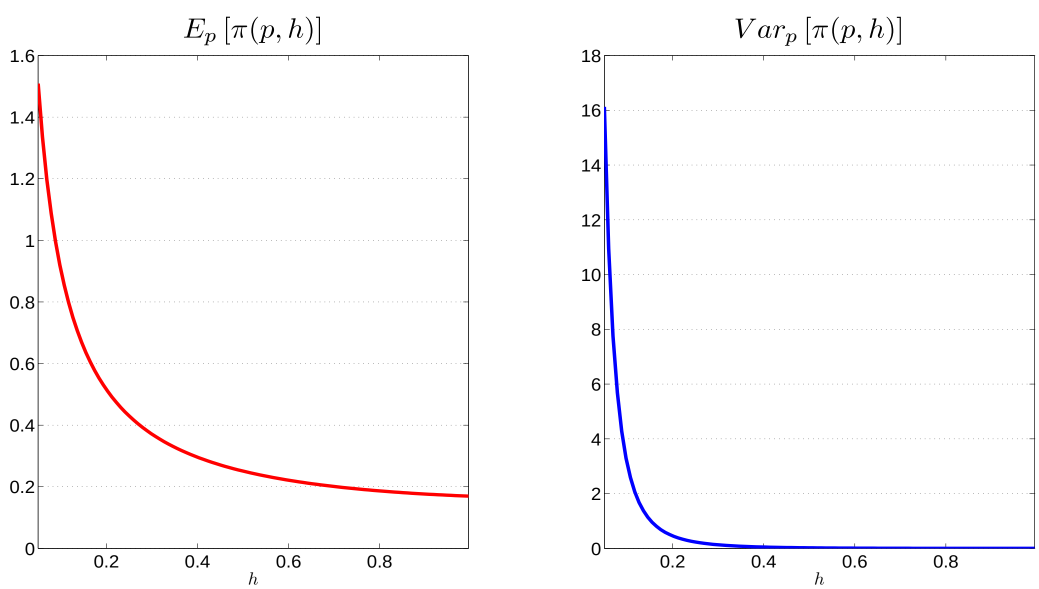


Figure 9: Endogenous Mean-Variance Trade-Off for a Hedging Policy.

Notes: *Parameters used* $p_L = 0.05$, $p_H = 4$, $\bar{p} = 2.43$ *and* $\alpha = 0.33$. *See Appendix E.1 for details.*

Assume a firm that lives for two periods, whose objective is to maximize dividend payments in the two periods. In the first period, she can only pay dividends by issuing debt. Let $Q(b', \sigma')$ be the price of a bond issued by a firm with debt b' and riskiness choice σ'. If the firm doesn't default in the second period, her dividends are the project pay-off net of debt payments, $a - b'$. If the firm defaults, dividend payments are zero. Let $E(b', \sigma')$ be the value to a firm for a given set of policies (b', σ'). Then

$$E(b', \sigma') = \{Q(b', \sigma')b' + \mathbb{E}_a [\max \{0, a - b'\} |\sigma']\} \tag{50}$$

Note that the firm will decide to default whenever $a < b'$. Then, the probability of default is $F(b', \sigma')$, where $F(\cdot)$ is the distribution of a given a firm's risk choice σ'.

I assume that lending is done by risk-neutral agents, with deep pockets and whose marginal utility of consumption in the second period is $\tilde{m}$. Then, the price of a bond issued by a firm that is choosing (b', σ') is as follows

$$Q(b', \sigma') = \tilde{m} [1 - F(b', \sigma')] \tag{51}$$

As in the larger model, I think of $\tilde{m}$ as an indicator of financial conditions in the economy since if directly affects bond prices. Equation (51) simply states that the lender will price the bond as his marginal value of consumption times the expected pay-off, which is the probability of no default.

Lets start by relaxing both the two key assumptions driving the σ' choice. Thus, assume that bond prices are only a function of debt b'.[86] Also, assume that there are no technological gains of increasing σ': $\mathbb{E}[a|\sigma] = 1$ for all σ'.[87]. Then, the value to the firm for a *given* policies (b', σ') is $E(b', \sigma') = Q(b')b' + \mathbb{E}_a [\max \{0, a - b'\} |\sigma']$. It can be shown that $\frac{\partial E(b', \sigma')}{\partial \sigma'} = \phi \left(\frac{-\ln b' + \sigma'^2}{\sigma'} \right) > 0$ for all $b' > 0$, where $\phi(\cdot)$ is the pdf of a standard normal.[88] Thus, for any positive amount of debt b', the firm finds it always profitable to increase σ' in the margin. This would clearly be an undesirable outcome, since firms would always choose the highest possible σ'. Furthermore, in this environment no positive amount of debt could be sustained.

The result discussed above is know in the literature as the "option value effect", and was first discussed by Jensen and Meckling (1976).[89] It is a consequence of the firm's pay-off ($\max \{0, a - b'\}$) being convex in a and thus it's expectation increasing in σ'. Allowing for the bond price $Q(\cdot)$ to depend on σ' introduces a trade-off: firms have the "option value effect" benefit of increasing σ', but also face the cost of decreasing

[86] This would be the case, for instance,if σ' is unobservable and no credible report can be made by the firm.

[87] For the $\mu(\cdot)$ used in the main text, this would be the case if $\varphi_1 = 0$ and $\varphi_2 = -\frac{1}{2}$.

[88] A derivation of this result is available upon request.

[89] See also Leland (1998)

their debt price. As I argue next, this trade-off is enough for the firm to take on lower σ', but may be too strong and yield $\sigma' \approx 0$.

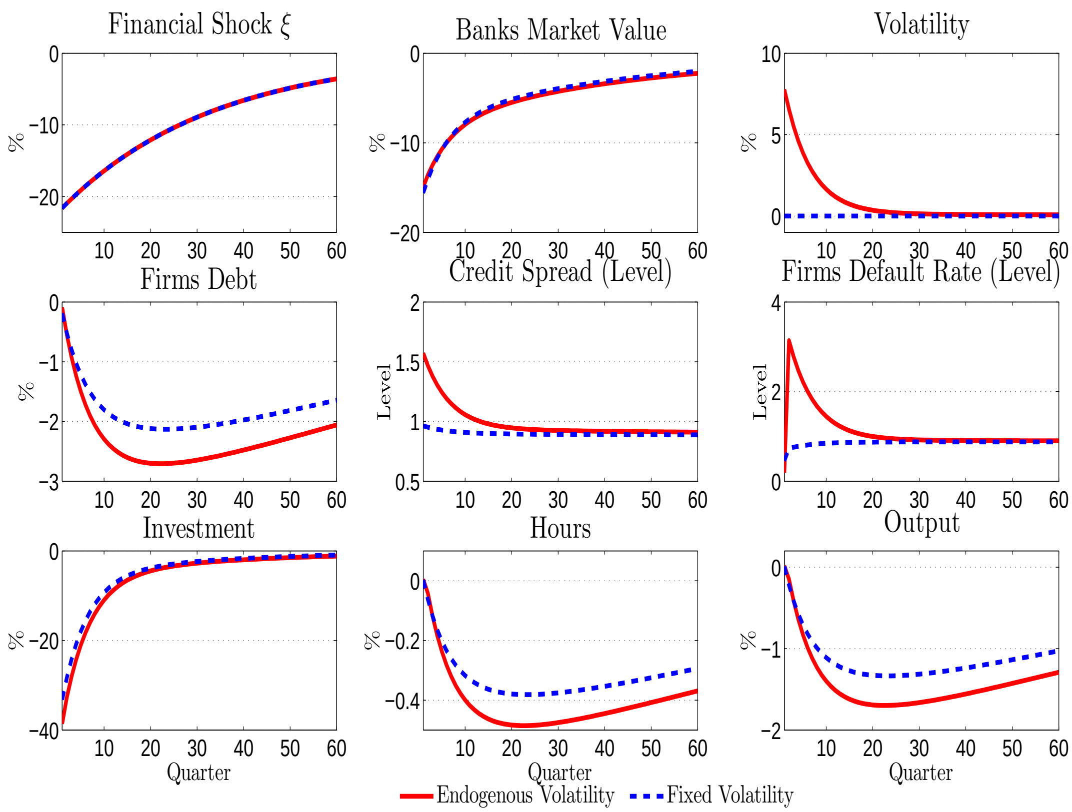
Assume now that the bond price depends on σ', but there are still no technological gains of increasing σ'. In this case, the value to a firm for a *given* policies (b', σ') is given by $E(b', \sigma') = Q(b', \sigma')b' + \mathbb{E}_a \left[\max\{0, a - b'\} | \sigma'\right]$. It can be shown that $\frac{\partial E(b',\sigma')}{\partial \sigma'} \propto (\tilde{m} - 1)\left[\ln b' - \frac{1}{2}\sigma'^2\right] + \left[\ln b' + \frac{1}{2}\sigma'^2\right]$.[90] In turn, as long as probabilities of default are below 50%, we have that $\ln b' < -\frac{1}{2}\sigma'^2$ and thus $\frac{\partial E(b',\sigma')}{\partial \sigma'} < 0$. Thus, the fact that the increase in risk σ' is priced in the debt, strongly disincentivizes risk taking and induce $\sigma' = 0$. In turn, the "option value effect" is only strong enough for default probabilities that are too high.

This last discussion provides the reason of why I need to assume $\mu(\cdot)$ increasing in σ. Once the bond price reflects the risk choice σ' on the margin,t The firm must have a benefit other than the "option value effect" to induce her to increase risk σ'. I think this discussion sheds light on how fragile the "option value effect" can be.

G Additional Figures

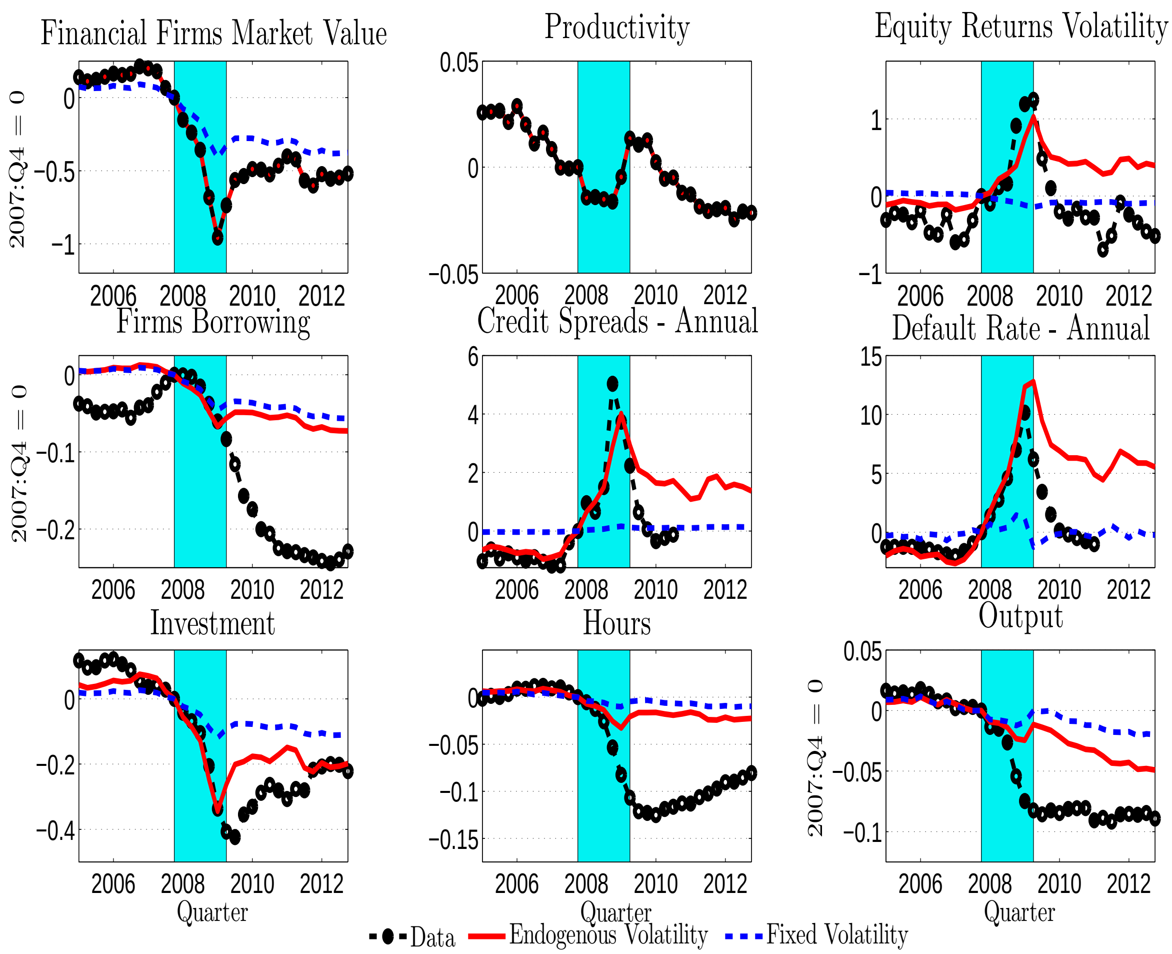
[90] A derivation of this result is available upon request.

Figure 10: Impulse response function to a negative financial shock ξ. Lower capital adjustment costs



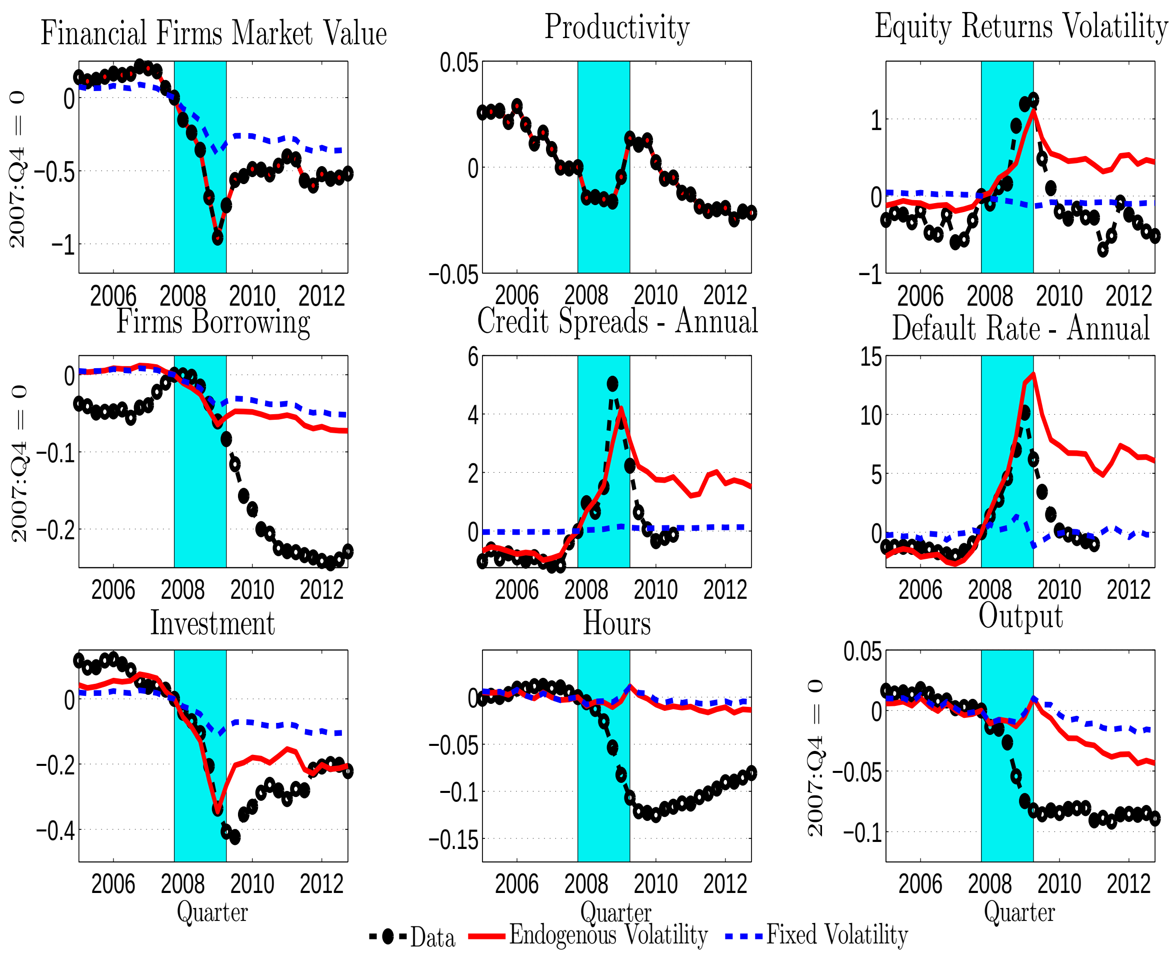
Notes: *Model response to an innovation in ξ that declines banks' market value by 50%. The red solid line corresponds to the benchmark model and the dashed blue line corresponds to a model with fixed volatility. Model with* ***lower capital adjustment costs****, $\phi_1 = 0.25$. See main text for details.*

Figure 11: 2007-2009 Crisis - Model Inference. Flexible Wages



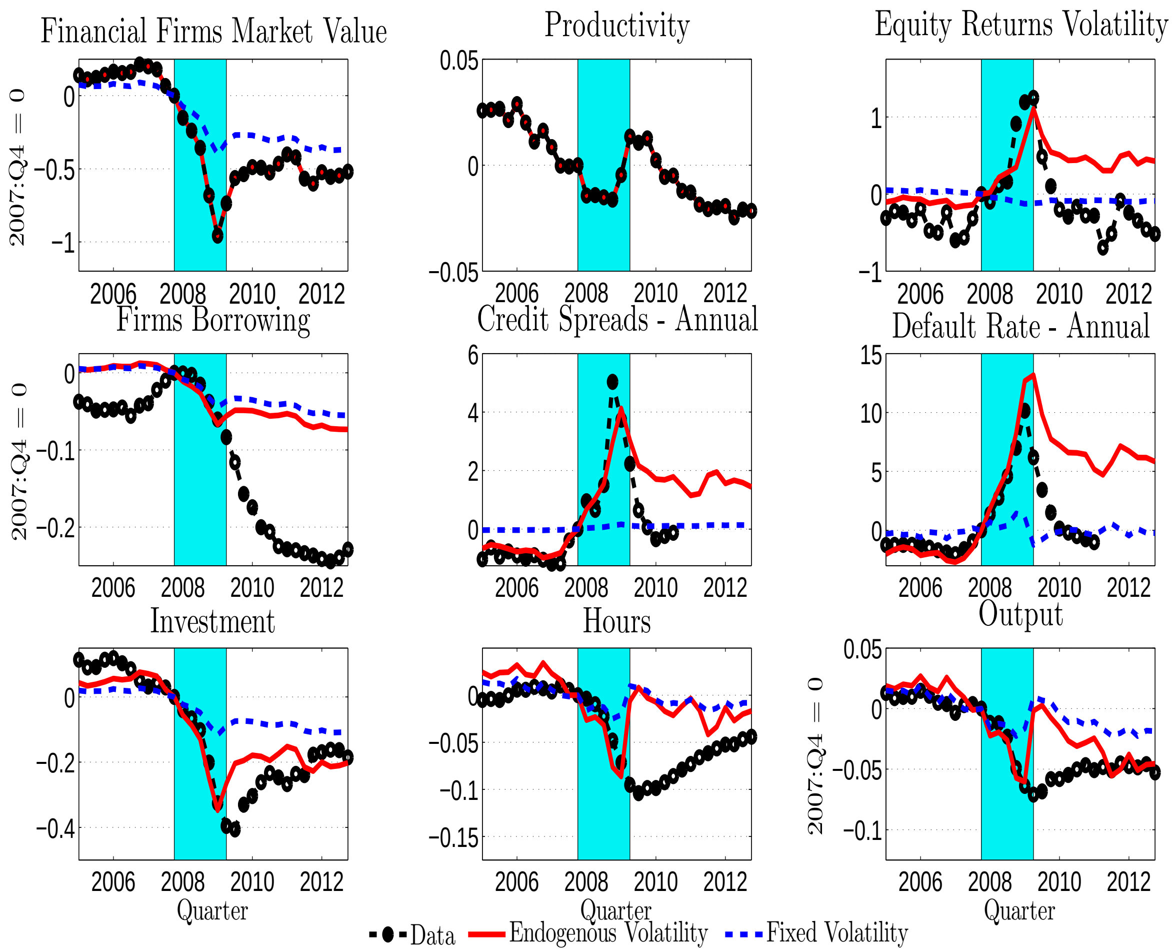
Notes: *Inference for period 1961:1-2012:4. Black circled lines correspond to data and red solid lines corresponds to the benchmark model. The blue dashed lines are the responses in the fixed volatility model to the shocks inferred in the benchmark model. All series normalized to 2007:Q4 = 0. See main text for data construction. Model with* ***flexible wages*** *and* ***working capital****.*

Figure 12: 2007-2009 Crisis - Model Inference. No Working Capital



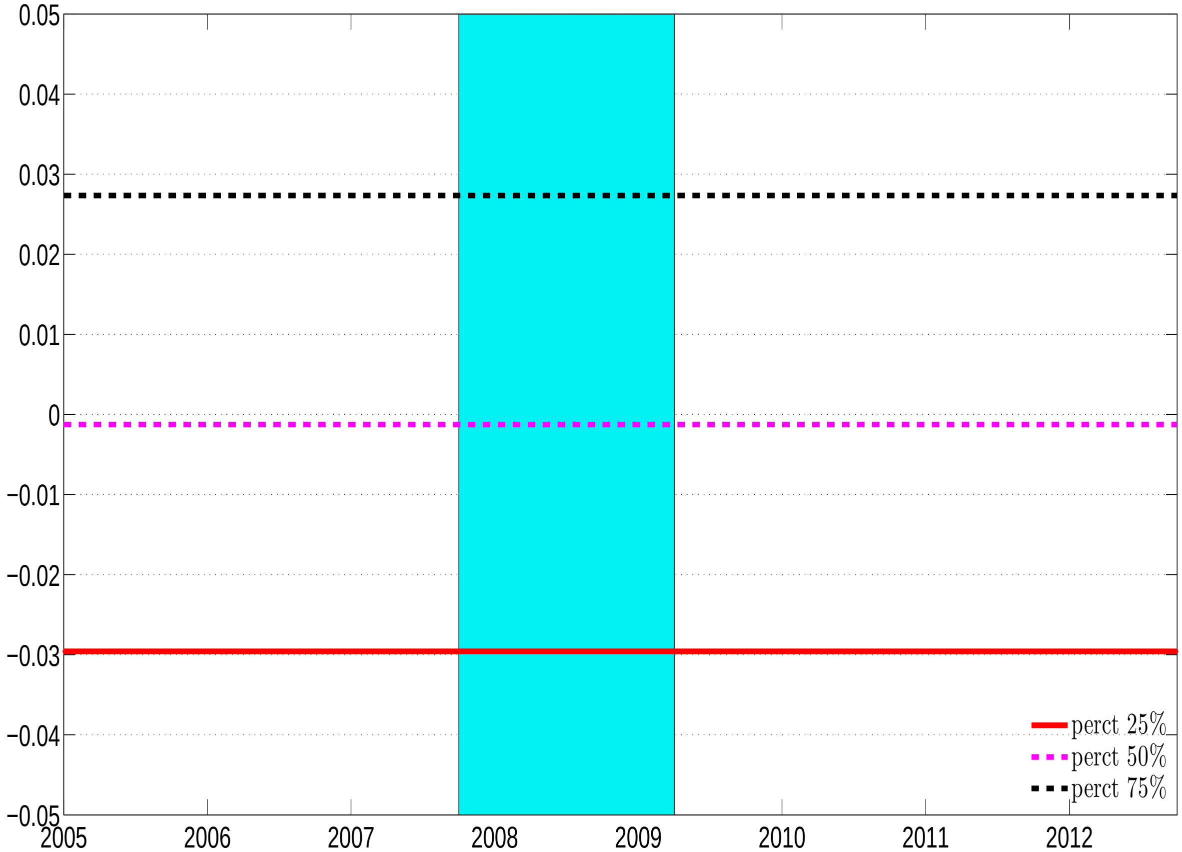
Notes: *Inference for period 1961:1-2012:4. Black circled lines correspond to data and red solid lines corresponds to the benchmark model. The blue dashed lines are the responses in the fixed volatility model to the shocks inferred in the benchmark model. All series normalized to 2007:Q4 = 0. See main text for data construction. Model with* ***sticky wages*** *and* ***no working capital****.*

Figure 13: 2007-2009 Crisis - Model Inference. Labor Force.



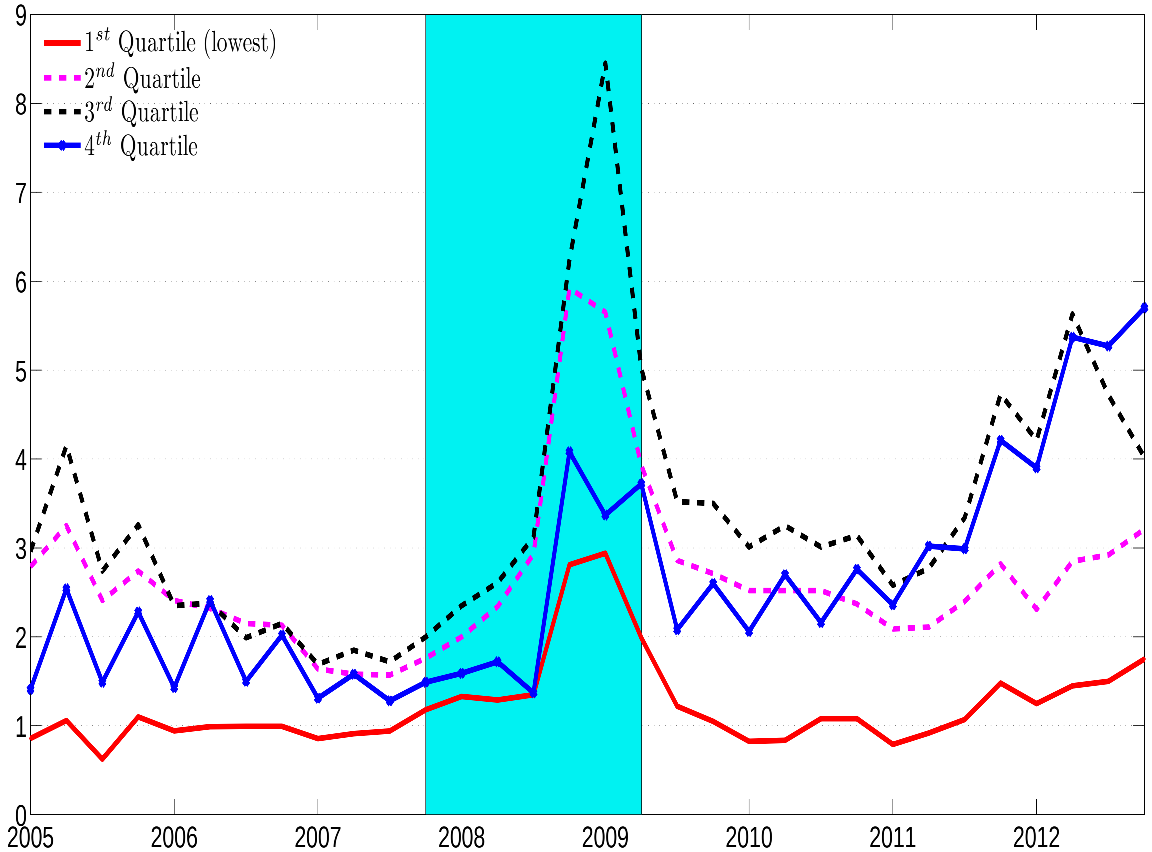
Notes: *Inference for period 1961:1-2012:4. Black circled lines correspond to data and red solid lines corresponds to the benchmark model. The blue dashed lines are the responses in the fixed volatility model to the shocks inferred in the benchmark model. All series normalized to 2007:Q4 = 0. See main text for data construction. Model with* ***sticky wages*** *and* ***working capital****. Output, investment and hours worked are normalized by labor force.*

Figure 14: Percentiles of Debt Growth during the 2007-2009 Crisis



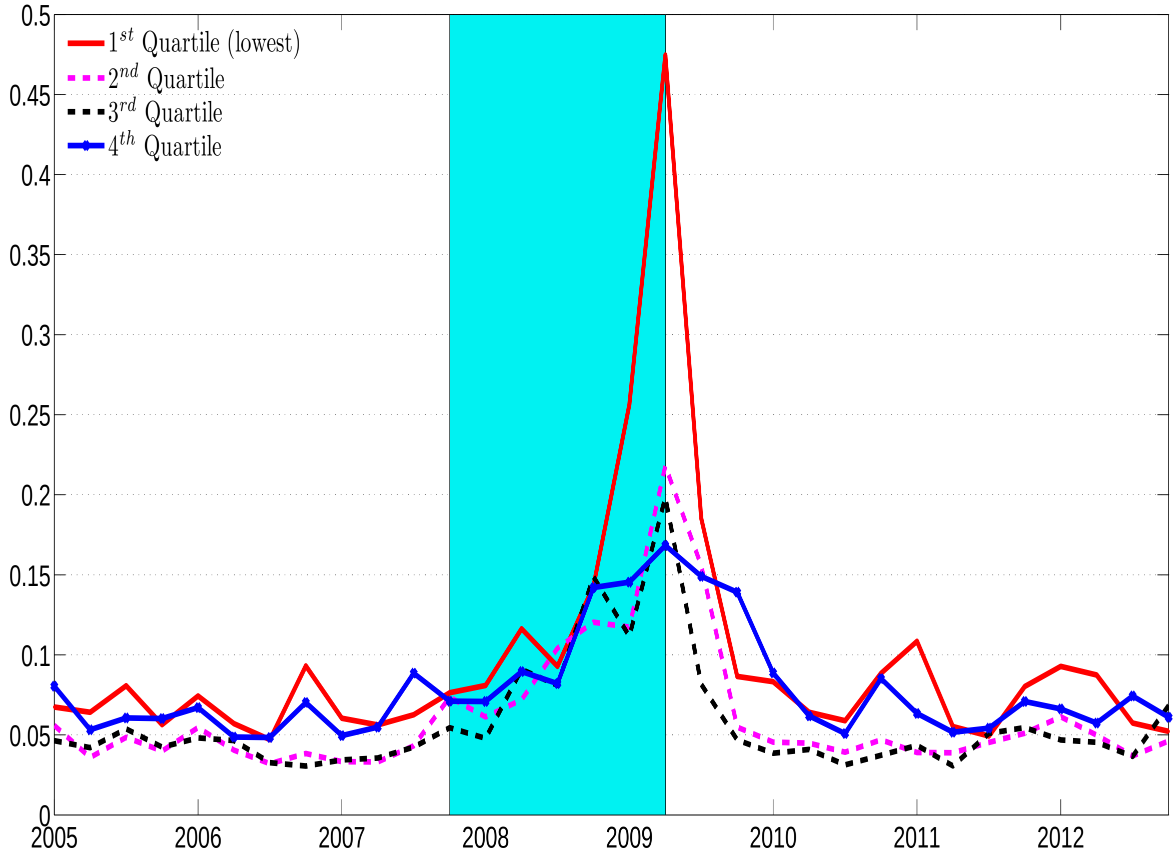
Notes: *Firms are divided into quartiles according to their average debt growth during 2007:Q4 to 2009:Q2. I keep only non-financial firms. Firms' debt comes from Compustat and is defined as total liabilities minus deferred tax liabilities. See the main text for more details*

Figure 15: Market Leverage by Quartiles of Debt Growth



Notes: *Firms are divided into quartiles according to their average debt growth during 2007:Q4 to 2009:Q2. I keep only non-financial firms. Market leverage is defined as firm's debt over firm's market value. Firms' debt comes from Compustat and is defined as total liabilities minus deferred tax liabilities. Firms' market value is also from Compustat. See the main text for more detials*

Figure 16: Equity Returns Volatility within Quartiles of Debt Growth - by quarter



Notes: *Firms are divided into quartiles according to their average debt growth* ***in each quarter****. I keep only non-financial firms. Firms' debt comes from Compustat and is defined as total liabilities minus deferred tax liabilities. Returns come from CRSP. See the main text for more details.*