Financial Crises and Endogenous Volatility

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Motivation: 2007-2009 Crisis

Facts during the 2007-2009 crisis

1. Large capital losses in the financial sector.
   - Market value of financial firms declined more than 50%.
2. A widespread increase in volatility for non-financial sector. Measures
   - Dispersion of equity returns for non-financial firms increased 300%.
3. A large contraction in economic activity.
   - GDP declined by 4% and investment by more than 30%.

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Questions:

1. What caused the increase in firms' volatility?
2. How was it related to the capital losses in the financial sector?

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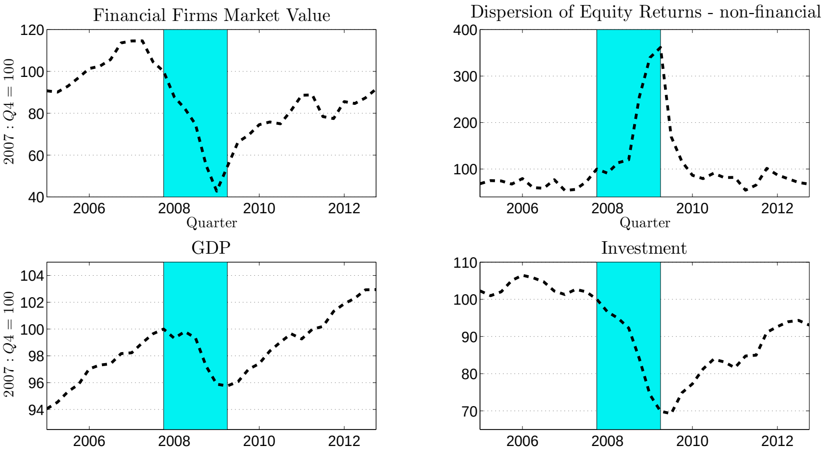

Notes: *Financial firms market value is outstanding shares times price of share for financial firms in CRSP. Equity returns volatility is the cross-sectional dispersion for non-financial firms in CRSP. GDP and Investment come from FRED. Quarterly data 2005:1 - 2012:4.*

Previous Work

1. Models with **frictions in intermediation**

2. Models with **stochastic volatility**

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   - Lending is constrained by banks' net worth.
   - Net worth losses contracts lending and economic activity.

   (Gertler & Kiyotaki, 2009)

2. Models with **stochastic volatility**

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   - Lending is constrained by banks' net worth.
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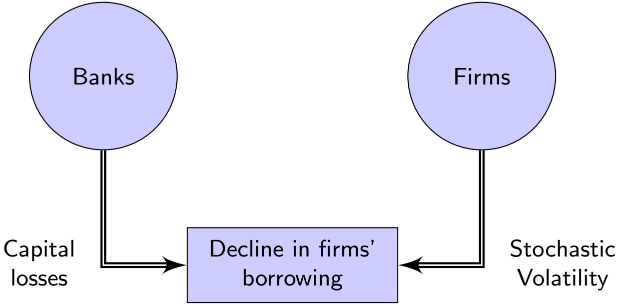
2. Models with **stochastic volatility**
   - Higher volatility increases firms' default risk.
   - This contracts lending and economic activity.

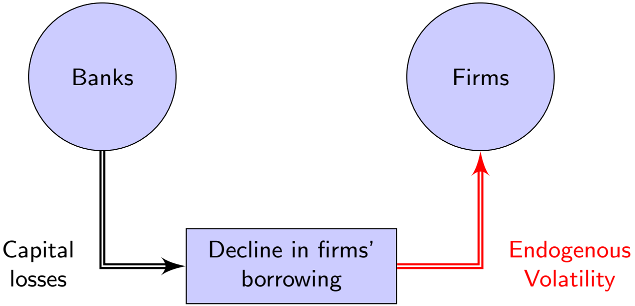
   (Arellano, Bai & Kehoe, 2012), (Christiano, Motto & Rostagno, 2014), (Gilchrist, Sim & Zakrajsek, 2013)

Previous Work



Question: link between banks' capital losses and firms volatility?

This Paper: Endogenous Volatility



Mechanism: firms' *volatility arises endogenously* due to financial disruptions.

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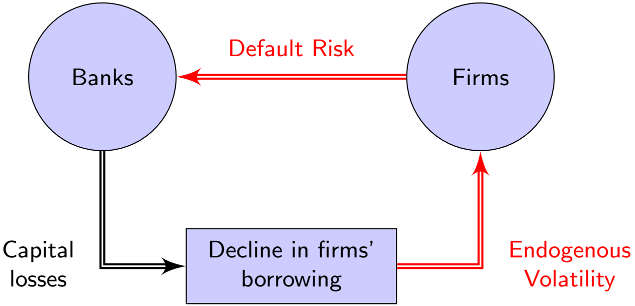

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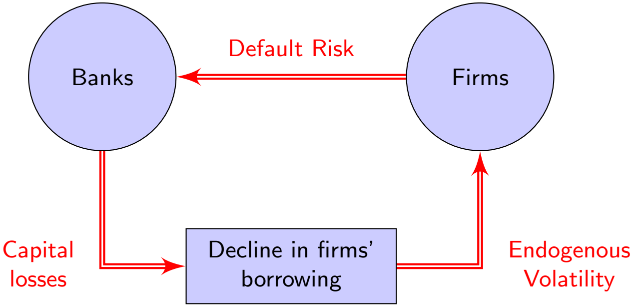

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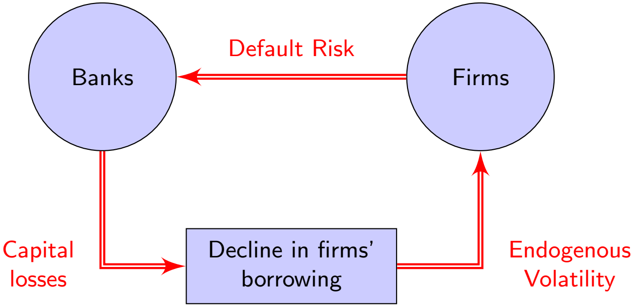

Mechanism: firms' *volatility arises endogenously* due to financial disruptions.

⇒ Evaluate mechanism during 2007-2009 crisis.

Mechanism: Key Ingredients

- Banks

- Firms

- Financial shock

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  - Leverage constraint: lending to firms is limited by banks' net worth

    $\Rightarrow$ banks' net worth losses decreases lending.

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  - Select riskiness of the projects they run.
  - Issue defaultable debt.
  - *Trade-off:* cost of borrowing increases with riskiness.

    high borrowing $\Rightarrow$ low risk-projects
    low borrowing $\Rightarrow$ high risk-projects.

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  - banks' net worth decreases $\Rightarrow$ lending decreases.

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- Financial shock
  - banks' net worth decreases $\Rightarrow$ lending decreases.
  - firms select riskier projects $\Rightarrow$ profits' volatility raises.

This paper: Results

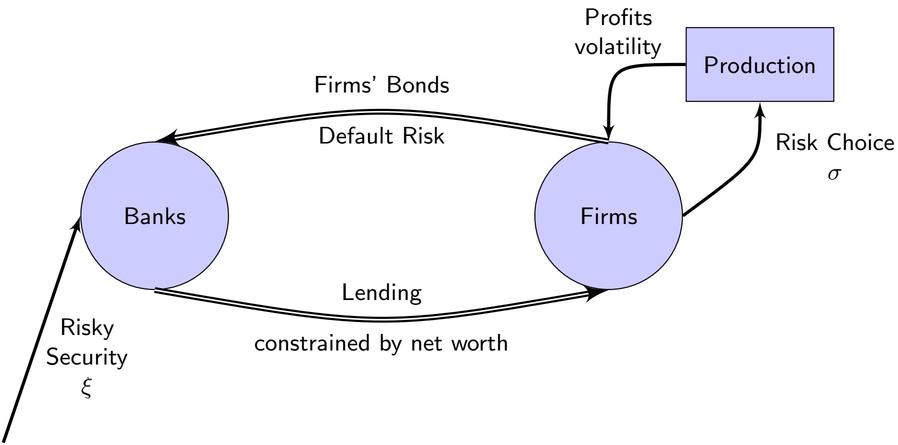
- Explore a new propagation mechanism for financial shocks.
- Banks' losses as observed during the 2007-2009 crisis induce
  - o Increase in dispersion of equity returns in line with evidence.
  - o Decline in investment of similar magnitude to data.
  - o Decline in output and hours worked as in data.
    (with **labor market** frictions)
- Endogenous volatility is key
  - Accounts for 70% of the drop in investment.
  - Accounts for 65% of the drop in output and hours worked.

Model

Environment

- **Demography:** *household*, *firms*, *banks* and *capital producers*.
- **Preferences:** $\mathbb{E}\sum_{t=0}^{\infty}\beta^t U(C_t, H_t)$
- **Technology:** $y_i = (a_i x k_i)^{\alpha}\, n_i^{1-\alpha}$.
- **Risk:** Firms **choose** next period distribution $a \sim F(\sigma)$
- **Firms**
  - **long-term debt**: a coupon c and **maturing** fraction λ.
  - have a **tax benefit** τ on coupon payments c.
  - upon **default**, a firm disappears.
- **Bankers**
  - **borrow** deposits from households, lend **long-term** to firms.
  - **risky** security with stochastic return ξ.
  - Face a **leverage constraint**.
- (x, ξ) follow Markov processes.

Model Flows


Profits
volatility
Production
Firms' Bonds
Default Risk
Banks
Firms
Risk Choice
σ
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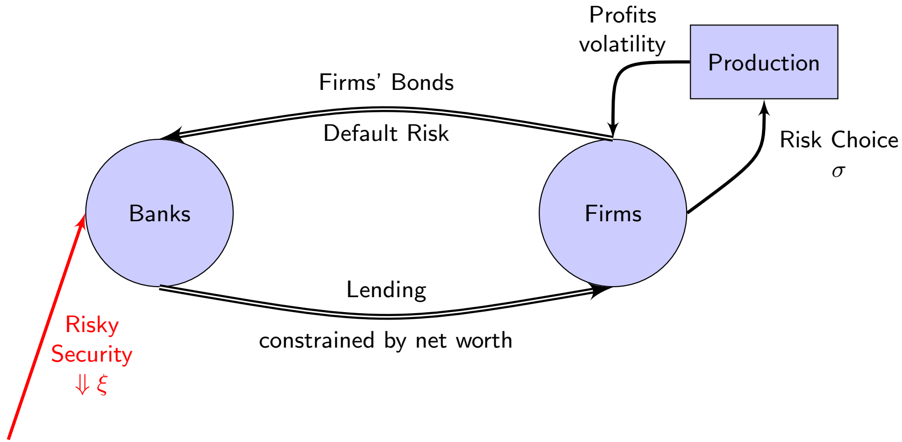
Model Flows

Household



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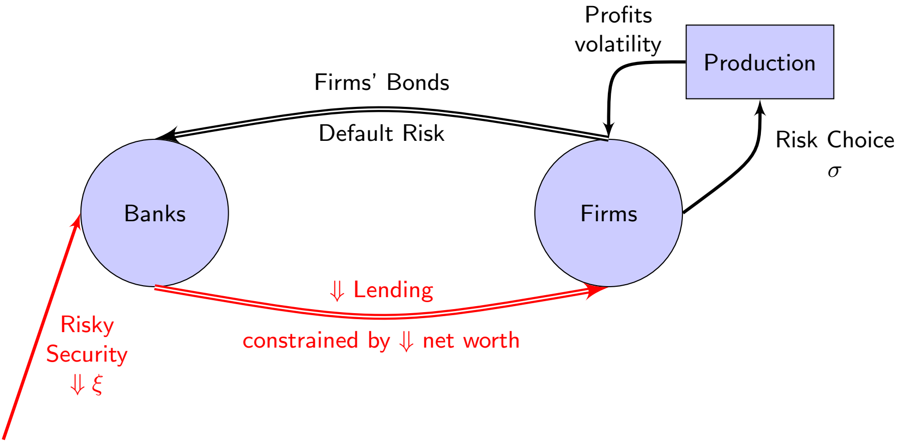
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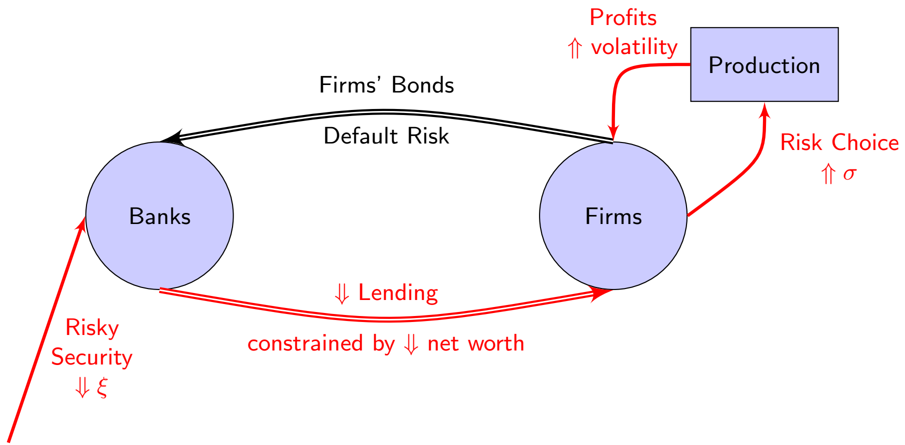

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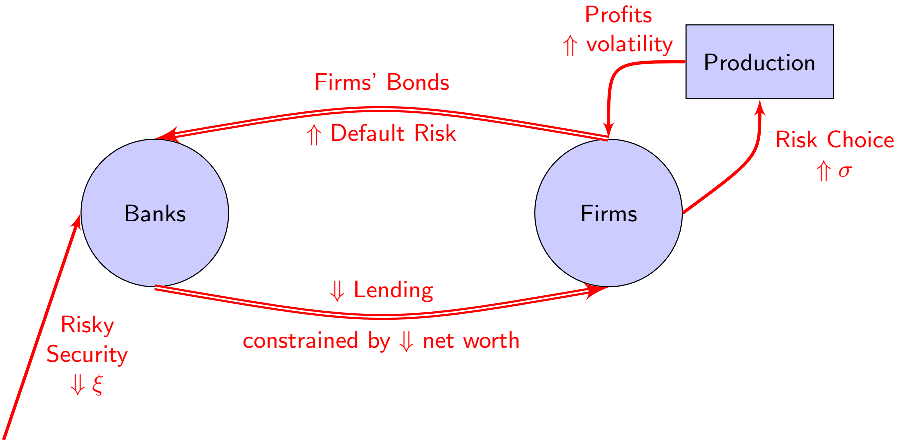

Timing

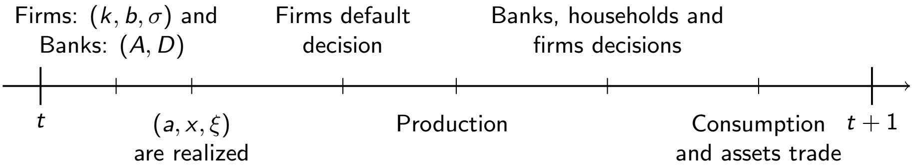

Figure: Timing within a period

Firms: Production

Static Problem If the firm did not default, she can hire labor and produce. Let S be the aggregate state of the economy

$$\Pi(a, k, S) = \max_n \left\{ (axk)^\alpha n^{1-\alpha} - w(S)n \right\}$$

Result

Profits are linear in capital

$$\Pi(a, k, S) = ak\pi(S)$$

Firms: Risk Choice

- Firm chooses a conditional distribution for the idiosyncratic shock

$$\ln a \sim \mathcal{N}(\mu(\sigma), \sigma^2)$$

- I assume

$$\mu(\sigma) = \mu_a + (\varphi_1 - \varphi_2\sigma)\,\sigma$$

  with $\varphi_1, \varphi_2 > 0$.

- σ must be chosen a period in advanced
  - *publicly observed*.

Firm cont'd

Let $E(a,b,k,S)$ be the value of a *non-defaulting* firm with productivity a, debt b and capital k when the aggregate state of the economy is S. Then

$$E(a,b,k,S) = \max_{d,k',b',\sigma'} \left\{ d + \mathbb{E}_{a',S'}\left[m(S,S') \max\left\{0, E(a',b',k',S')\right\} | \sigma', S \right] \right\}$$

subject to

$$\mathcal{P} = (1-\tau)\left[ak\pi(S) - cb\right] - \lambda b$$

$$d + Q^k(S)k' \leq \mathcal{P} + Q^k(S)k(1-\delta) + Q(l',\sigma',S)\underbrace{\left[b' - (1-\lambda)b\right]}_{\text{new debt } \Delta b'}$$

where $l' = b'/k'$ is *leverage* next period.

Firm cont'd

Lemma

- *Firm's value function is linear in* k*:* $E(a, b, k, S) = \mathcal{P} + e(I, S)k$.
- *Policies are:*

$$
\begin{aligned}
k'(a, b, k, S) &= \boldsymbol{\iota}(I, S)k \\
b'(a, b, k, S) &= \boldsymbol{\ell}'(I, S)k'(a, b, k, S) \\
\sigma'(a, b, k, S) &= \boldsymbol{\sigma}'(I, S)
\end{aligned}
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- *Default follows a threshold decision* $\underline{\mathbf{a}}(l,S)$
  - *Leverage makes default more likely:* $\frac{\partial \underline{\mathbf{a}}(l,S)}{\partial l} > 0$

more

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Banks

A period in the life of a Banker:

1. Repays **deposits** to households and collect **returns** on previous investment.

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   - **Firms' bonds** $b'(l', \sigma')$ at price $Q(l', \sigma', S)$.
   - **Risky-security** A' at a price $Q^A(S)$.
   - **Deposits** D' from households at price $Q^D(S)$.

$$\int Q(l', \sigma', S) b(l', \sigma') dl' d\sigma' + Q^A(S) A' \leq N + Q^D(S) D'$$

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4. Face an incentive constraint.
   - Can run away with a fraction θ of bank's assets.
   - Portfolios are **constrained** so that they don't run away in equilibrium.

$$V(N,S) \geq \theta \left[\int Q(l',\sigma',S) b(l',\sigma') dl' d\sigma' + Q^A(S) A' \right]$$

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5. Returns

$$N' = \underbrace{\left[\xi' + Q^A(S')\right]}_{\text{risky-asset return}} A' - D'$$
$$+ \int \underbrace{\left[1 - F(\underline{a}(I', S'), \sigma')\right]}_{\text{non-default rate}} \underbrace{\left[(c + \lambda) + (1 - \lambda) Q(\boldsymbol{\ell}'(I', S'), \boldsymbol{\sigma}'(I', S'), S')\right]}_{\text{payment upon non-default}} b(I', \sigma') dI' d\sigma'$$

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5. Returns
6. A new bank replaces the exiting one with equity $\bar{\omega}N$

Banks

Let $V(N, S)$ be the value of a bank with wealth N when the aggregate state of the economy is S. Then

$$V(N,S) = \max_{\{b(l',\sigma')\}_{\{l',\sigma'\}},A',D'} \left\{ \mathbb{E}_{S'} \left[m(S,S') \left\{ (1-\psi)N' + \psi V(N',S') \right\} | S \right] \right\}$$

subject to

$$\underbrace{\int Q(l',\sigma',S)b(l',\sigma')dl'd\sigma'}_{\text{lending to firms}} + \underbrace{Q^A(S)A'}_{\text{risky-securities purchases}} \leq N + \underbrace{Q^D(S)D'}_{\text{new deposits}}$$

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Banks cont'd

Lemma

- *Bank's value function is linear in net worth* $V(N, S) = v(S)N$.
- *Portfolio decisions are linear in net worth.*
- *Incentive constraint yields a leverage constraint.*
- *Bank's* **stochastic discount factor** *is given by*

$$\tilde{m}(S, S') = m(S, S') \frac{(1 - \psi) + \psi v(S')}{\kappa(S) + \theta \eta(S)}$$

where $\kappa(S)$ *and* $\eta(S)$ *are the multipliers on the budget and incentive constraints, respectively.*

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more

Government and Aggregates

Standard Elements:

- Household: $w(S) = -U_H(S)/U_C(S)$
- Capital producers: $Q^k(S) = 1/\Phi'(I(S)/K)$

Government:

- Balanced budget constraint

$$T_H(S) = \tau \int \{-\lambda c b_i + a_i k_i \pi(S)\} \mathbb{I}(a_i > \underline{\mathbf{a}}(I,S))di + (1-\Gamma)F(\underline{\mathbf{a}}(I,S),\sigma)Q^k(S)K$$

Aggregates:

- Risky-Asset: $\int A'(N_i, S)di = \bar{A}$
- Feasibility: $Y(S) + \xi\bar{A} = C(S) + I(S)$
- State of the economy: $S = (x, \xi, \sigma, I, K, D)$

Equilibrium

Definition

A **recursive competitive equilibrium** is given by:

+ Value functions: firm E, bank V, and household V^H;
+ Policies: firm $\{d, b', k', \sigma'\}$; bank $\{b'(l', \sigma'), A', D'\}$; and household $\{C, H, D'\}$;

such that, given prices $\{w, Q(l', \sigma'), Q^D, Q^k, Q^A\}$:

- Agents optimize and achieve values E, V, and V^H.
- Markets clear:
  - **Labor market**: $\int n(a_i, k_i, S)di = H(D, S)$,
  - **Bonds market**: $\int b'(a, b, k, S) = \int b(l', \sigma', N_i, S)di \quad \forall l', \sigma'$,
  - **Deposits market**: $\int D'(N_i, S)di = D'(D, S)$,
  - **Risky market**: $\int A'(N_i, S)di = \bar{A}$,
  - **Goods market**: $Y(S) + \xi\bar{A} = C(S) + I(S)$,

Model Characterization

Bond Pricing and the Effect of Financial Shocks

- The **price of a bond** indexed by (l', σ') is given by

$$Q(l', \sigma', S) = \mathbb{E}_{S'}\Big[\tilde{m}(S, S')\left[1 - F\left(\underline{\mathbf{a}}(\cdot), \sigma'\right)\right]\left[(c+\lambda) + (1-\lambda)Q(\boldsymbol{\ell}'(\cdot), \boldsymbol{\sigma}'(\cdot), S')\right]|S\Big]$$

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  - $v(S)$ is banks' marginal value of wealth.
  - $\kappa(S)$ and $\eta(S)$ are multipliers on budget and incentive constraints.

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A decline in banks' net worth $\Rightarrow$ decline in $\tilde{m}(S, S')$ $\Rightarrow$ decline in $Q(l', \sigma', S)$

Lower bond prices, lower leverage and higher risk

- Marginal benefit of increasing l'

$$\overbrace{Q(l',\sigma',S)}^{\text{price of debt}}$$

Decline in $Q(l',\sigma',S) \Rightarrow$ decrease in leverage l'

more | Effect λ and τ

Lower bond prices, lower leverage and higher risk

- Marginal benefit of increasing l'

$$\overbrace{Q(l', \sigma', S)}^{\text{price of debt}}$$

Decline in $Q(l', \sigma', S) \Rightarrow$ decrease in leverage l'

more | Effect λ and τ

- Marginal cost of increasing σ'

$$-\overbrace{\frac{\partial Q(l', \sigma', S)}{\partial \sigma'}}^{\text{effect on price}} \overbrace{[l' - (1-\lambda) l]}^{\text{new debt}}$$

Decline in leverage $l' \Rightarrow$ increase in σ'

more

Lower bond prices, lower leverage and higher risk

- Marginal benefit of increasing l'

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Decline in leverage $l' \Rightarrow$ increase in σ'

more

Decline in banks net worth $\Rightarrow$ decline $Q(l', \sigma', S) \Rightarrow$ decline l', increase σ'

Amplification

- Bank's net worth

$$
\begin{aligned}
N &= \left[\xi + Q^A(S)\right] A - D \\
&+ \left[1 - F\left(\underline{\mathbf{a}}(\cdot), \sigma\right)\right] \left[(c+\lambda) + (1-\lambda) Q\left(\boldsymbol{\ell}'(\cdot), \boldsymbol{\sigma}'(\cdot), S\right)\right] b
\end{aligned}
$$

Amplification

- Bank's net worth

$$\begin{aligned} N &= \left[\Downarrow \xi + Q^{A}(S)\right] A - D \\ &+ \left[1 - F\left(\underline{\mathbf{a}}(\cdot), \sigma\right)\right] \left[(c+\lambda) + (1-\lambda) Q\left(\boldsymbol{\ell}'(\cdot), \boldsymbol{\sigma}'(\cdot), S\right)\right] b \end{aligned}$$

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    - Decline in $\Downarrow N$

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  + Propagation:
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    - Decline in bond prices $\Downarrow Q(\cdot)$.
  - Amplification
    - Further decline in N

Model Evaluation

Calibration

- Utility: $U(C,N) = \left[C - \chi \frac{N^{1+1/\eta}}{1+1/\eta}\right]^{1-\gamma} \frac{1}{1-\gamma}$
- Capital production: $\Phi(I/K) = \phi_0 (I/K)^{1-\phi_1} + \phi_2$

Calibrated Parameters

Parameter	Value	Target/Source
β	0.99	Discount Factor
(γ, η, χ)	$(1, 2.5, 1.75)$	Standard
(ϕ_0, ϕ_1)	$(0.02, 2.5)$	Guvenen (2009)
(δ, ϕ_2)	$(0.025, 0.003)$	Investment-to-capital $\approx$ 1.7%
$1-\alpha$	0.64	Labor share
(ρ_x, σ_x)	$(0.95, 0.0075)$	Fernald (2012)
Γ	0.12	Bernanke, Gertler & Gilchrist (1999)
τ	0.4	Firms' debt to GDP $\approx$ 3.45
c	0.075	Firms' leverage $\approx$ 45% (Book Value)
λ	1/24	Average maturity of 6 years
$\mathbb{E}(\xi)$	0.4	Financial Sector/GDP $\approx$ 12%
(ρ_ξ, σ_ξ)	$(0.97, 0.036)$	CRSP Financial Firms Market Value
ψ	0.975	Banks' life of 10 years
θ	0.125	Annual Credit Spread of $\approx$ 0.9%
$\bar{\omega}$	0.5	Banks Leverage $\approx$ 8

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1. Mean-variance relation: $\mu(\sigma) = \mu_a + (\varphi_1 - \varphi_2\sigma)\sigma$. Pick (φ_1, φ_2) to
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   - match **variance** over time of the cross-sectional dispersion of equity returns.

2. A "disaster" absorbing draw: $a \sim p \ln \mathcal{N}(\mu(\sigma), \sigma^2) + (1-p)0$
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Calibrated Parameters

Parameter	Value	Target/Source
p	0.99875	Annual Default Rate 1%
(φ_1, φ_2)	$(0.62, 1.17)$	$\mathbb{E}(\sigma_t) \approx 0.4$ and **variance** ≈ 0.33
μ_a	-0.14	$\mathbb{E}(a) \approx 1$

- Linear solution method
  - incentive constraint is always binding

Business Cycle Moments

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Business Cycle Moments

Crisis Experiment

Exercise: Model response to a financial shock ξ that induces a 50% decline in banks market value

Question: How much endogenous volatility amplifies?

- Compare to the same economy with fixed volatility.

Crisis Experiment

Funding | Capital Costs | Productivity

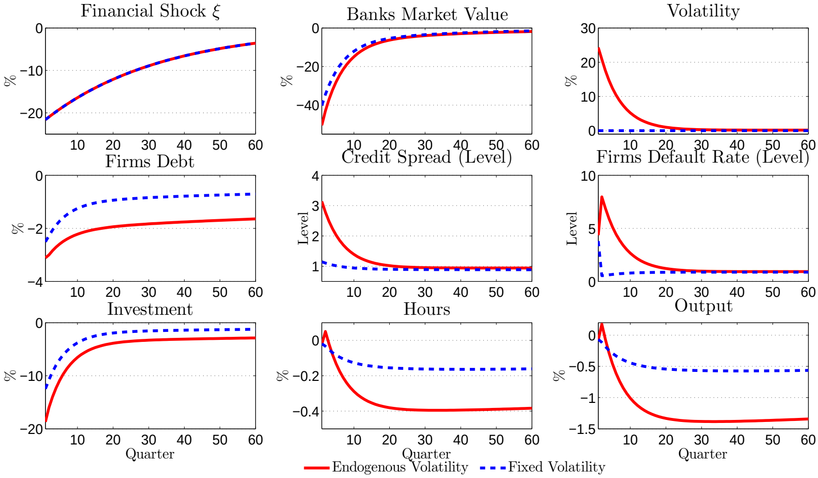

Figure: Model Response to a 50% decline in banks market value

Crisis Experiment

Results:

- Financial shocks increases volatility.
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1. **Real sticky wages** more
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   - Monopolistically supply a differentiated type of labor.
   - Unions adjust wages with probability $\theta_w = 0.75$ every period.
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2. **Working capital** at firm level more
   - Firms have to borrow a fraction $\theta^L = 0.67$ of the wage bill.*
   - One period debt, no default risk.
   - Banks lend the working capital resources.

*(short-term liabilities to assets $\approx$ 25%, non-financial firms, Compustat)

2007-2009 Crisis

Question: Can the model replicate the events of the 2007-2009 crisis?

Exercise:

- Feed the model with observed data (linear detrend) Data
    - Banks Market Value (CRSP, Financial Firms)
    - Productivity (Fernald, 2012)

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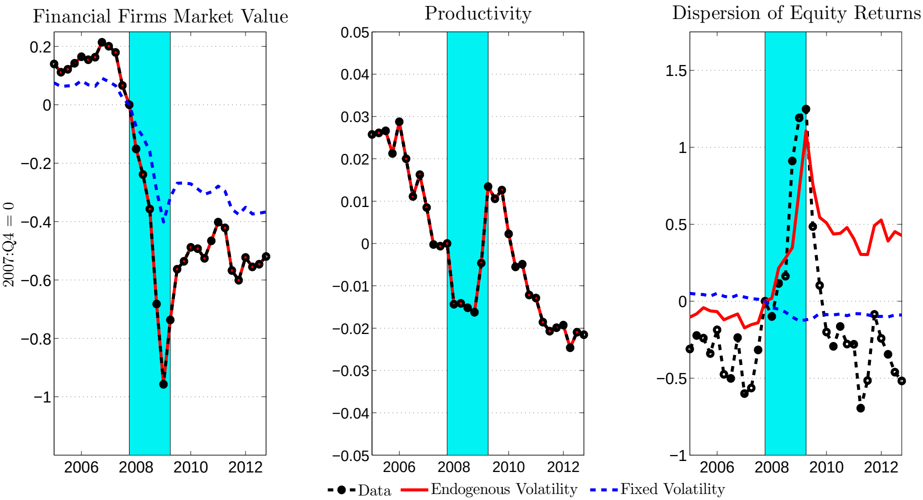
- What does the model predict for other variables?
- How important is the endogenous volatility ?
  - o Compare to the same economy with fixed volatility.

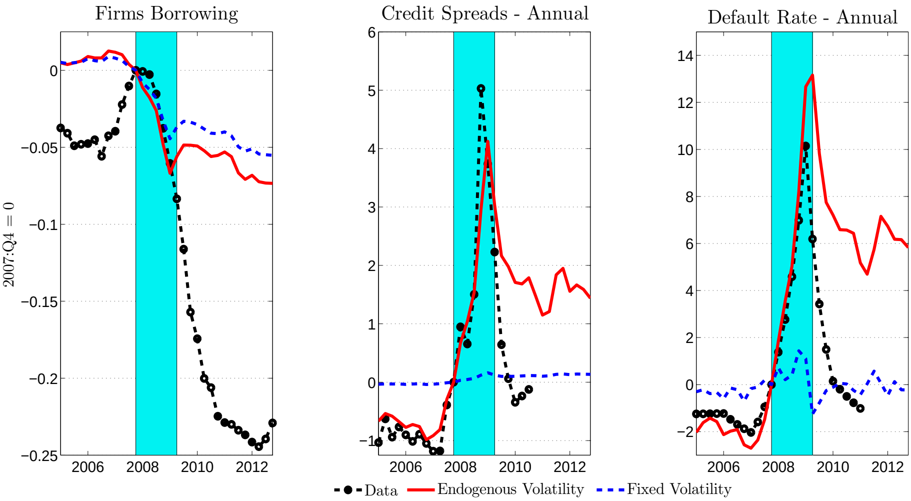
Volatility Computation

2007-2009 Crisis



Note: *Inference for period 1961:1-2012:4. All series normalized to 2007:Q4 = 0. Model with* **sticky wages** *and* **working capital**.

2007-2009 Crisis



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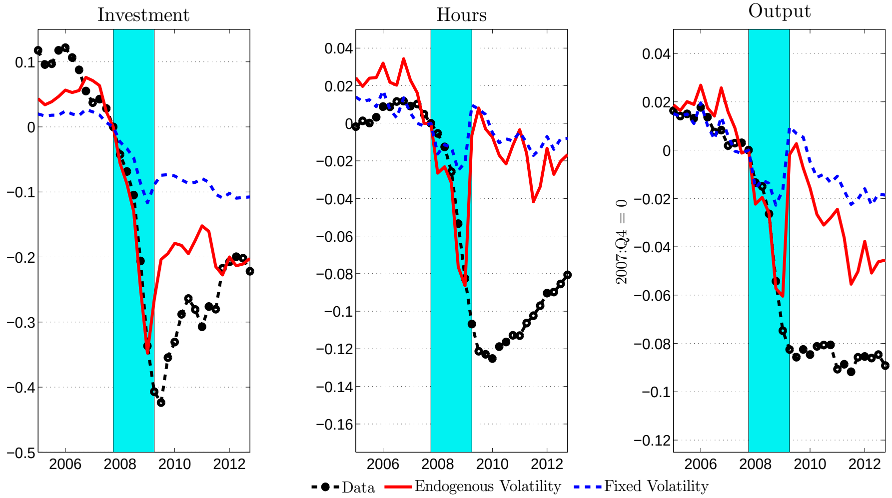

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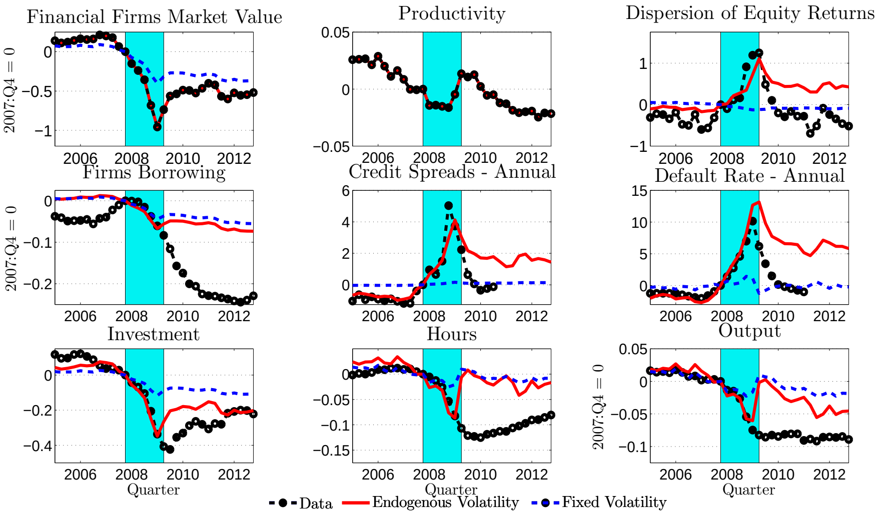
2007-2009 Crisis

No Frictions | more on prices | more on inference | Evidence



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Conclusions

- Facts during the 2007-2009 crisis
  - Large capital losses in the financial sector.
  - Increase in firms' volatility.
  - Contraction in economic activity.
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  - **Key idea**: endogenously higher volatility due to poor lending conditions.
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- A step towards incorporating foundations of volatility in DSGE models.

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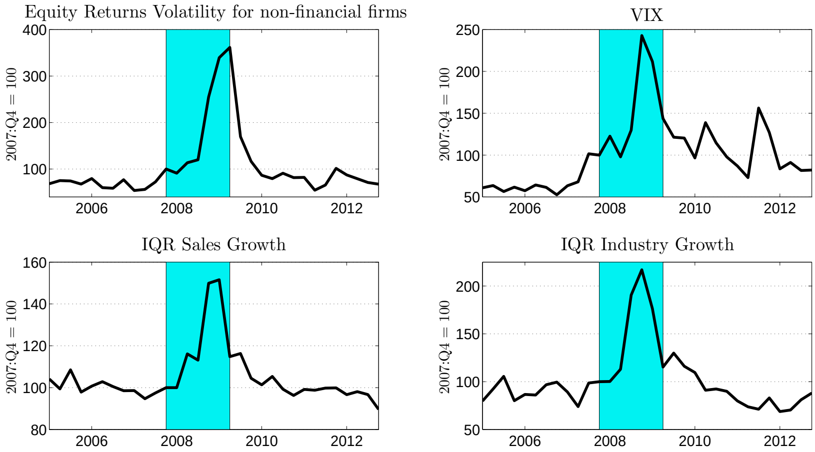
Thank you!!!

Motivation: 2007-2009 Crisis

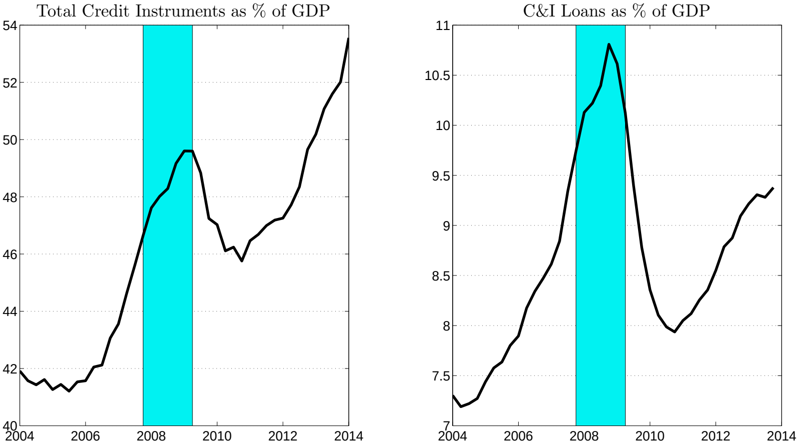
Return



Notes: *Equity returns volatility is the cross-sectional dispersion for non-financial firms in CRSP. VIX is the quarterly average for the S&P 500 market index. Sales growth is for non-financial firms on Compustat. Industry growth is from FRB industry database. Quarterly data 2005:1 - 2012:4.*

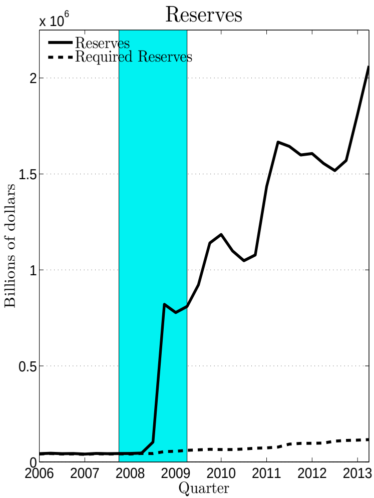
Motivation: A large decline in lending

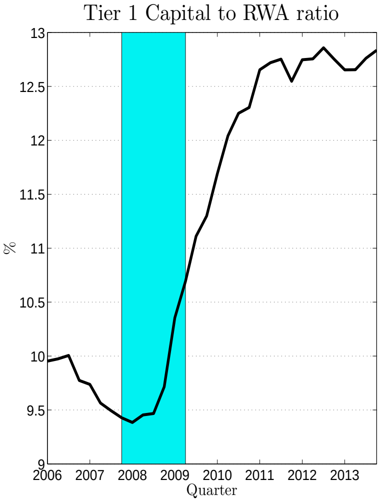
Return



Notes: *Total Credit Instruments comes from Table Z.1 from Flow of Funds. C&I Loans comes from Table H.8 from Flow of Funds. Loans correspond to non-financial corporate lending from commercial banks. GDP is in current dollars.*

Motivation: Lack of willingness to lend? Return





Notes: *Reserves are from Flow of Funds for Commercial Banks. Banks capital measures come from FDIC for Commercial Banks.*

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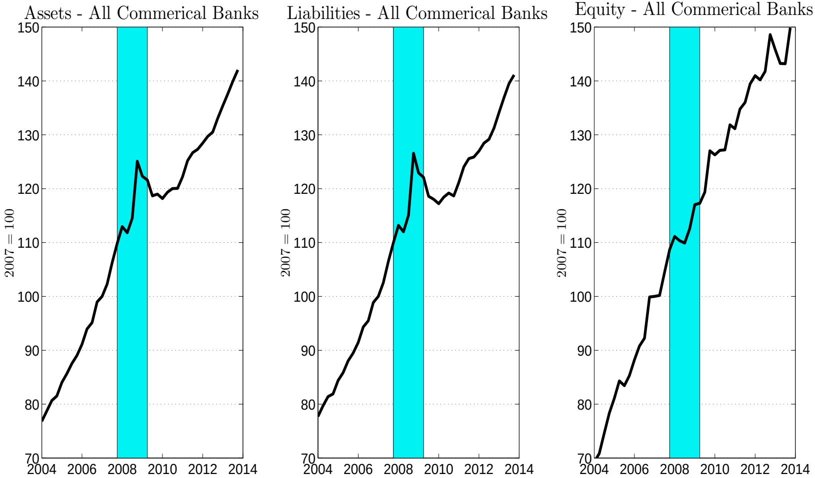
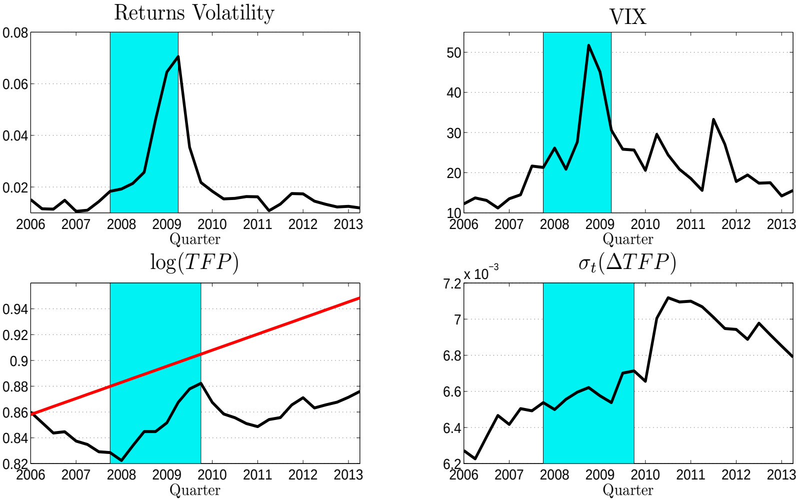


Notes: *Assets, Liabilities and Equity are nominal and book value. Data comes from FDIC for Commercial Banks.*

Motivation

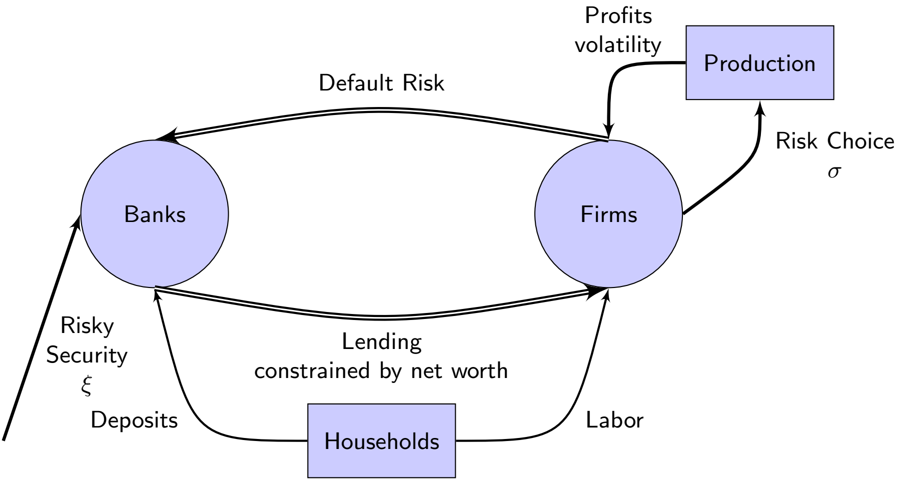
Return



Notes: *Returns volatility is computed from CRSP firms. VIX is from Yahoo! Finance. TFP comes from Fernald (2012), the red line is linear trend in logs.*

Model Flows

Return



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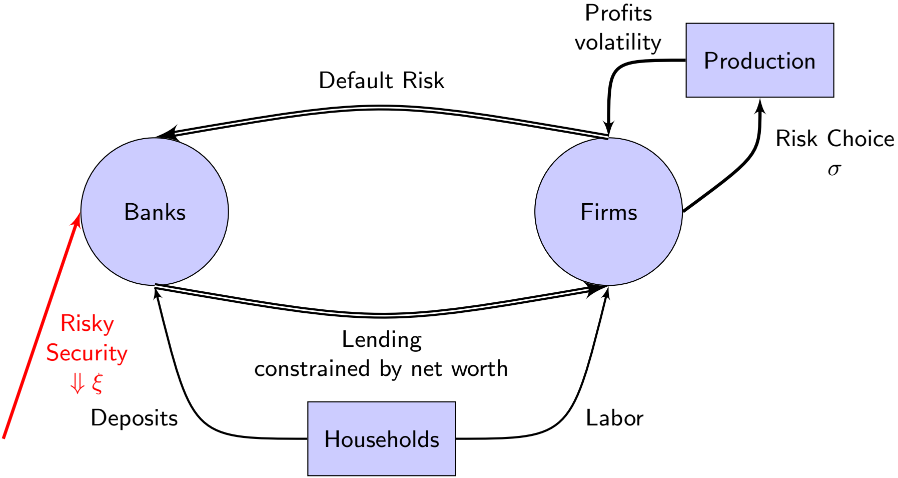

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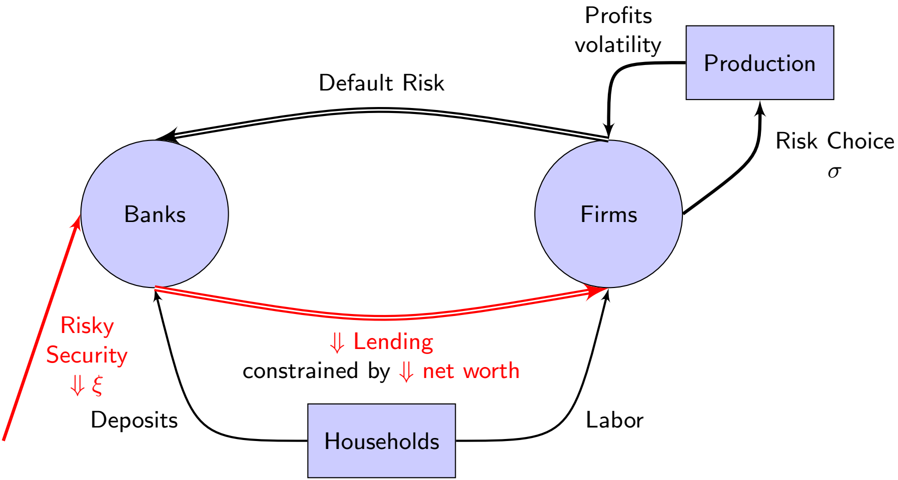

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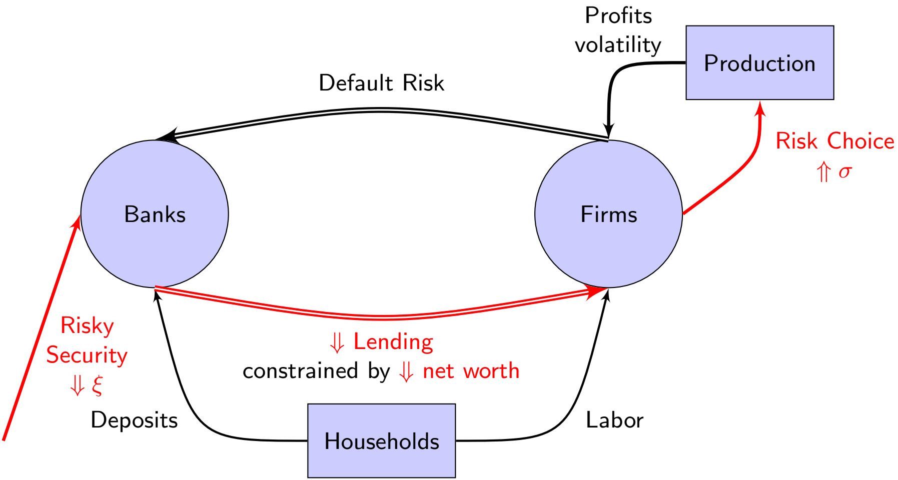
Profits
volatility
Production
Default Risk
Risk Choice
$\Uparrow \sigma$
Banks
Firms
Risky
Security
$\Downarrow \xi$
$\Downarrow$ Lending
constrained by $\Downarrow$ net worth
Deposits
Labor
Households

Model Flows

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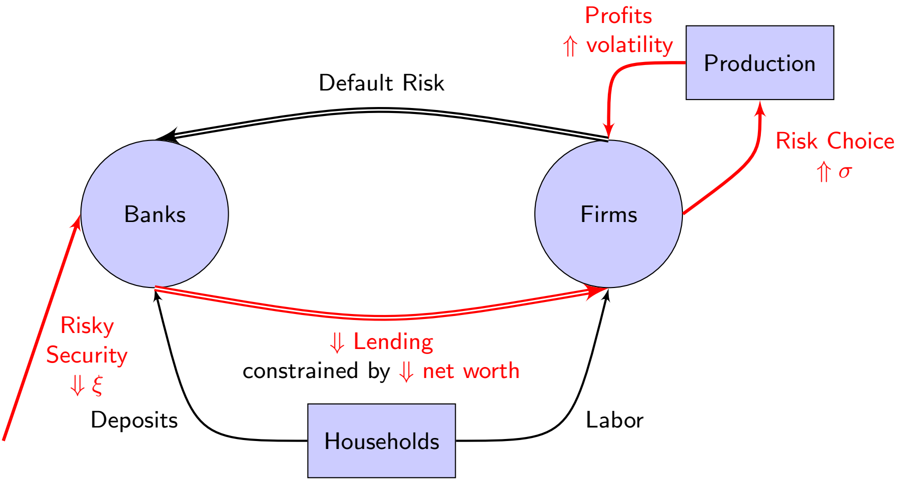

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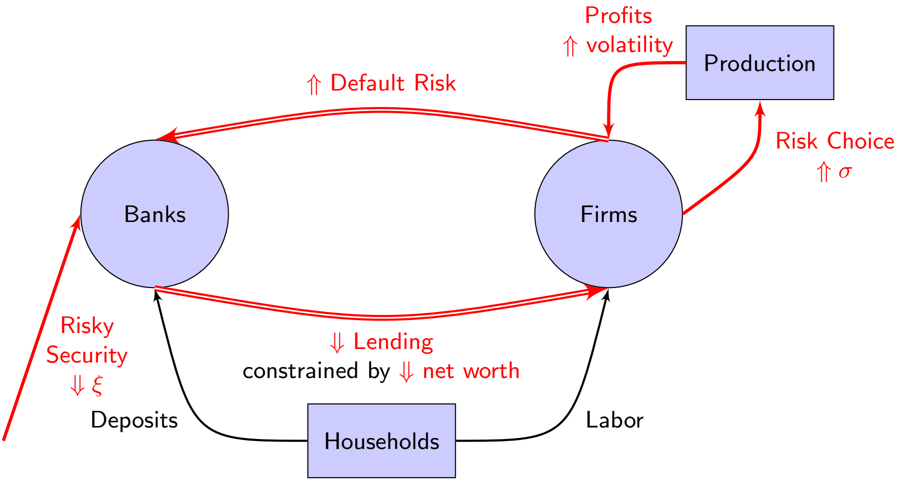

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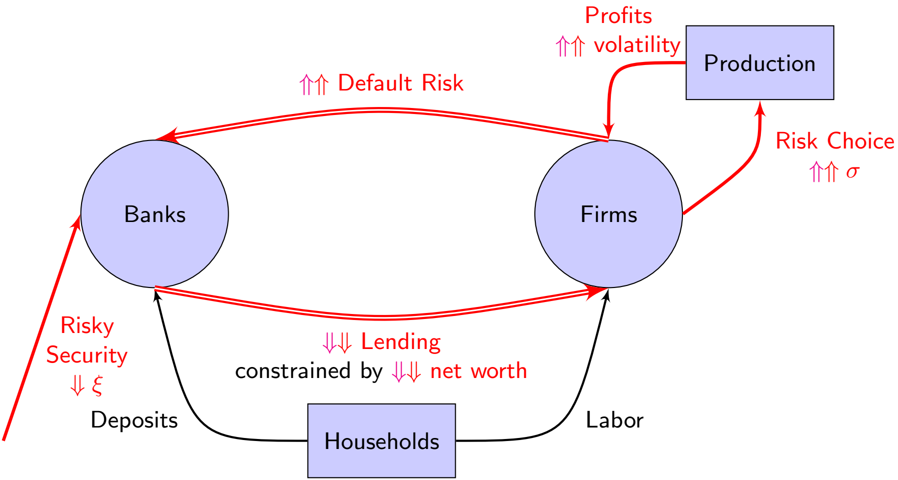

Firm cont'd - Lemma Return

Lemma

- *Firm's value function is linear in k: $E(a,b,k,S) = \mathcal{P} + e(I,S)k$.*
- *Policies are:*

$$\begin{aligned} k'(a,b,k,S) &= \boldsymbol{\iota}(I,S)k \\ b'(a,b,k,S) &= \boldsymbol{\ell}'(I,S)k'(a,b,k,S) \\ \sigma'(a,b,k,S) &= \boldsymbol{\sigma}'(I,S) \end{aligned}$$

- *Default follows a threshold decision $\underline{\mathbf{a}}(I,S)$:*

$$\underline{\mathbf{a}}(I,S) = \frac{1}{(1-\tau)\pi(S)}\Big[\{(1-\tau)c+\lambda\}I - e(I,S)\Big]$$

- *The value of installed capital $e(I,S)$ is given as*

$$e(I,S) = \underbrace{Q^k(S)(1-\delta)}_{\text{assets value}} - \underbrace{(1-\lambda)Q(\boldsymbol{\ell}'(I,S),\boldsymbol{\sigma}'(I,S),S)I}_{\text{liabilities value}}$$

Firm cont'd - Lemma Return

Lemma

- *Firm's value function is linear in* k*:* $E(a,b,k,S) = \mathcal{P} + e(I,S)k$.
- *Policies are:*

$$\begin{aligned} k'(a,b,k,S) &= \boldsymbol{\iota}(I,S)k \\ b'(a,b,k,S) &= \boldsymbol{\ell}'(I,S)k'(a,b,k,S) \\ \sigma'(a,b,k,S) &= \boldsymbol{\sigma}'(I,S) \end{aligned}$$

- *Default follows a threshold decision* $\underline{\mathbf{a}}(I,S)$*:*

$$\underline{\mathbf{a}}(I,S) = \frac{1}{(1-\tau)\pi(S)}\left[\{(1-\tau)c+\lambda\}\,I - e(I,S)\right]$$

- *The value of installed capital* $e(I,S)$ *is given as*

$$e(I,S) = \underbrace{Q^k(S)(1-\delta)}_{\text{assets value}} - \underbrace{(1-\lambda)Q(\boldsymbol{\ell}'(I,S),\boldsymbol{\sigma}'(I,S),S)I}_{\text{liabilities value}}$$

Firm cont'd - Lemma Return

Lemma

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Firm cont'd - Lemma Return

Lemma

- *Firm's value function is linear in k: $E(a,b,k,S) = \mathcal{P} + e(l,S)k$.*
- *Policies are:*

$$\begin{aligned} k'(a,b,k,S) &= \boldsymbol{\iota}(l,S)k \\ b'(a,b,k,S) &= k'(a,b,k,S) \\ \sigma'(a,b,k,S) &= \end{aligned}$$

- *Default follows a threshold decision :*

$$\underline{\mathbf{a}}(l,S) = \frac{1}{(1-\tau)\pi(S)}\Big[\{(1-\tau)c+\lambda\}\, l - e(l,S)\Big]$$

- *The value of installed capital $e(l,S)$ is given as*

$$e(l,S) = \underbrace{Q^k(S)(1-\delta)}_{\text{assets value}} - \underbrace{(1-\lambda)Q(\boldsymbol{\ell}'(l,S),\boldsymbol{\sigma}'(l,S),S)l}_{\text{liabilities value}}$$

Banks cont'd

Return

Lemma

Banks value function is linear and given by $V(N,S) = v(S)N$, where $v(S)$ is recursively defined as

$$v(S) = \frac{1}{Q^D(S)} \frac{\mathbb{E}_{S'}\left[m(S,S')\left\{(1-\psi)+\psi v(S')\right\}|S\right]}{1-\mu(S)}$$

where $\mu(S)$ is the multiplier on the incentive constraint. Finally, the incentive constraints reads

$$\frac{\int Q(l',\sigma',S)b(l',\sigma')dl'd\sigma' + Q^A(S)A'}{N} \leq \frac{v(S)}{\theta}$$

Capital Producers Return

- Continuum of firms under perfect competition
- Maximize static profits

$$\Pi^k(S) = \max_i \left\{ Q^k(S) \underbrace{\Phi\left(\frac{i}{K}\right) K}_{\text{new capital}} - i \right\}$$

where $\Phi' > 0$ and $\Phi'' < 0$.

- In equilibrium

$$Q^k(S) = \frac{1}{\Phi'\left(\frac{I(S)}{K}\right)}$$

- **Result:** price of capital $Q^k(S)$ increases with investment $I(S)$.

Household Return

Let $V^H(D,S)$ be the value of a household with D deposits when the aggregate state of the economy is S. Then

$$
\begin{aligned}
V^H(D,S) &= \max_{C,N,D'} \left\{ U(C,N) + \beta \mathbb{E}_{S'} \left[V^H(D',S') | S \right] \right\} \\
\text{subject to} & \\
C + Q^D(S)D' &\leq D + w(S)N - T_H(S) + d^F(S) + d^B(S) + \Pi^k(S) \\
d^F(S) &= \int \{ d_i \mathbb{I}(a_i > \underline{\mathbf{a}}(I,S)) + (1-\Xi) k_i \mathbb{I}(a_i \leq \underline{\mathbf{a}}(I,S)) \} \, di \\
d^B(S) &= (1-\psi)\Omega(S) - (1-\psi)\bar{\omega}\Omega(S) \\
S' &= \Gamma(S)
\end{aligned}
$$

Firm's leverage choice

Return

Leverage choice:

- Marginal benefit of increasing l'

$$\overbrace{Q(l',\sigma',S)}^{\text{price of debt}}$$

- Marginal cost of increasing l'

$$-\overbrace{\frac{\partial Q(l',\sigma',S)}{\partial l'}}^{\text{effect on price}}[l'-(1-\lambda)l]$$

$$+\mathbb{E}_{S'}\left[m(S,S')\left[1-F\left(\underline{\mathbf{a}}(l',S'),\sigma'\right)\right]\overbrace{\left[(\tilde{c}+\lambda)+(1-\lambda)\frac{\partial e(l',S')}{\partial l'}\right]}^{\text{effect on firm's value}}|S\right]$$

where $\tilde{c}=(1-\tau)c$ and $e(l,S)$ is the value of installed capital.

Firm's leverage choice

Return

Leverage choice:

- Marginal benefit of increasing l'

$$\overbrace{Q(l', \sigma', S)}^{\text{price of debt}}$$

- Marginal cost of increasing l'

$$-\overbrace{\frac{\partial Q(l', \sigma', S)}{\partial l'}}^{\text{effect on price}}[l' - (1-\lambda)l]$$

$$+\mathbb{E}_{S'}\left[m(S,S')\left[1 - F\left(\underline{\mathbf{a}}(l', S'), \sigma'\right)\right]\overbrace{\left[(\tilde{c}+\lambda) + (1-\lambda)\frac{\partial e(l', S')}{\partial l'}\right]}^{\text{effect on firm's value}}|S\right]$$

where $\tilde{c} = (1-\tau)c$ and $e(l, S)$ is the value of installed capital.

Firm's risk choice

Return

Risk choice:

- Marginal benefit of increasing σ'

$$\frac{\partial}{\partial \sigma'} \mathbb{E}_{S'} \left[m(S,S') \int_{\underline{\mathbf{a}}(l',S')}^{\infty} \overbrace{\left[\underbrace{\pi(S')a - (\tilde{c}+\lambda)\, l'}_{\mathcal{P}} + e(l',S') \right]}^{\text{effect on productivity + ``option value'' effect}} dF(a,\sigma') | S \right]$$

where $\tilde{c} = (1-\tau)c$ and $e(l,S)$ is the value of installed capital.

- Marginal cost of increasing σ'

$$-\overbrace{\frac{\partial Q(l',\sigma',S)}{\partial \sigma'}}^{\text{effect on price}} \overbrace{[l' - (1-\lambda)l]}^{\text{new debt}}$$

Firm's risk choice

Return

Risk choice:

- Marginal benefit of increasing σ'

$$\frac{\partial}{\partial \sigma'} \mathbb{E}_{S'} \left[m(S, S') \int_{\underline{a}(I', S')}^{\infty} \overbrace{\left[\underbrace{\pi(S')a - (\tilde{c} + \lambda) I'}_{\mathcal{P}} + e(I', S') \right]}^{\text{effect on productivity + ``option value'' effect}} dF(a, \sigma') | S \right]$$

where $\tilde{c} = (1 - \tau)c$ and $e(I, S)$ is the value of installed capital.

- Marginal cost of increasing σ'

$$- \overbrace{\frac{\partial Q(I', \sigma', S)}{\partial \sigma'}}^{\text{effect on price}} \overbrace{[I' - (1 - \lambda)I]}^{\text{new debt}}$$

Effect of λ and τ Return

- Assume that banks' incentive constraint never binds ($\mu(S) = 0 \quad \forall S$)
- Optimal leverage policy, for a given σ' and ι, is given by

$$\ell'(l, S) = \tau c \frac{\mathbb{E}_{S'}\left[m(S, S')\left[1 - F(\underline{\mathbf{a}}(\ell'(l, S), S'), \sigma')\right] | S\right]}{-\partial Q(\ell'(l, S), \sigma', S)} + \frac{1-\lambda}{\iota} l$$

- No corporate tax ($\tau = 0$) implies no debt.
- Long term debt ("low" λ) adds persistence to leverage.

Business Cycle Moments

Return

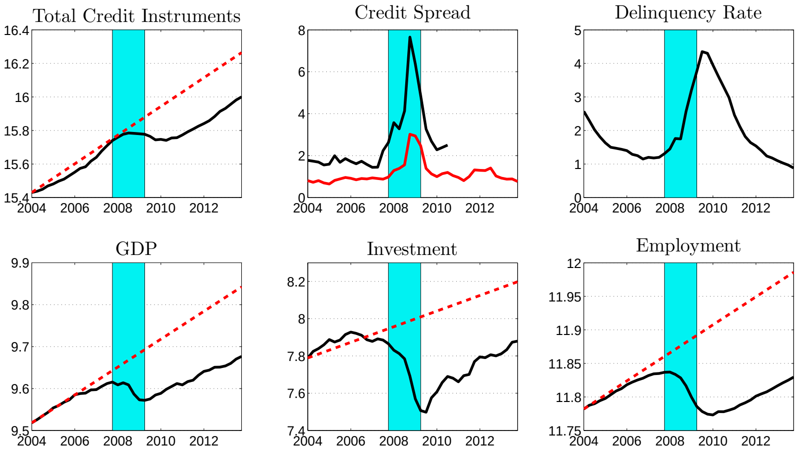
Table: Business Cycle Moments

x	$\sigma(x)/\sigma(GDP)$		$corr(x, GDP)$		$corr(x_t, x_{t-1})$	
	Data	Model	Data	Model	Data	Model
Investment	2.94	2.8	0.78	0.70	0.95	0.93
Debt	3.70	1.20	0.13	0.92	0.99	0.99
Leverage	1.33	1.17	0.11	−0.84	0.59	0.98
Returns Volatility	10.31	11.10	−0.10	−0.19	0.76	0.82
Credit Spread	9.92	27.13	−0.54	−0.22	0.93	0.84

Notes: *GDP and investment in 2009 chained dollars and deflated by working age population (OECD). Debt is total credit instruments for non-financial corporate business in the US (Flow of Funds). Leverage is for non-financial firms on Compustat (book value). Credit spreads are Baa - Aaa. Returns volatility is for non-financial firms on CRSP. All variables at quarterly frequency and computed as difference of a linear trend in logs. See paper for more details.*

2007-2009 Crisis

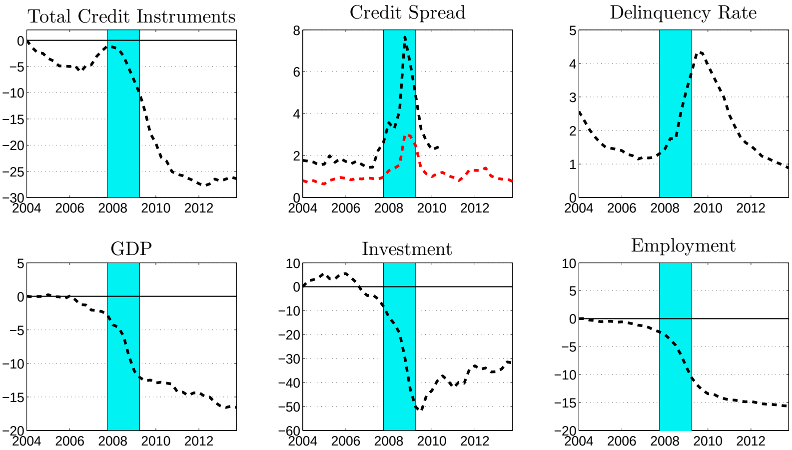
Return



Notes: *Total Credit Instruments comes from Table Z.1 from Flow of Funds for non-financial firms. Employment is total non-farm payroll. GDP, Investment and Employment come from FRED. The trend is computed for the period 1952:1 - 2005:4. All in logs*

2007-2009 Crisis

Return



Notes: *Total Credit Instruments comes from Table Z.1 from Flow of Funds for non-financial firms. Employment is total non-farm payroll. GDP, Investment and Employment come from FRED. The trend is computed for the period 1952:1 - 2005:4. All in logs*

Crisis Experiment - Funding

Return

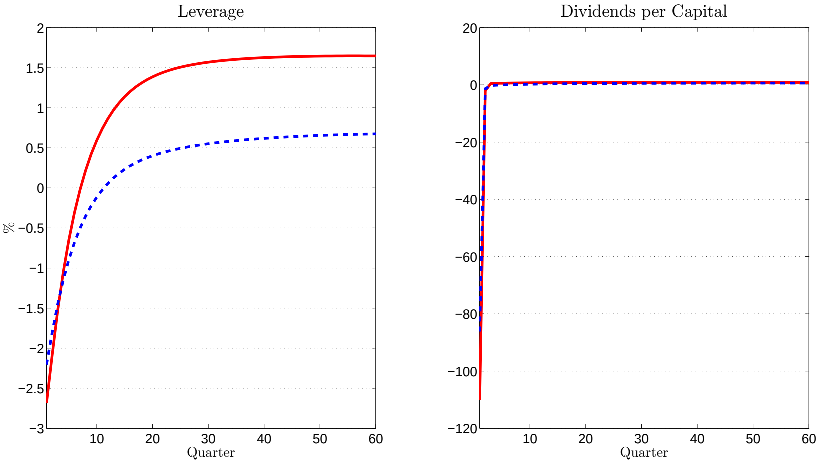


Figure: Model response to a 50% decline in banks market value

Crisis Experiment with low capital adjustment cost

Return

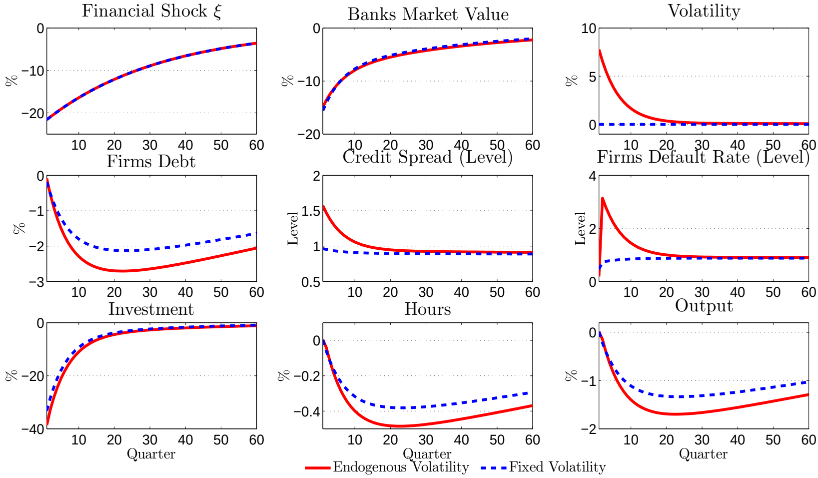


Figure: Model response to a financial shock ξ.

Model Response - Productivity Shock

Return

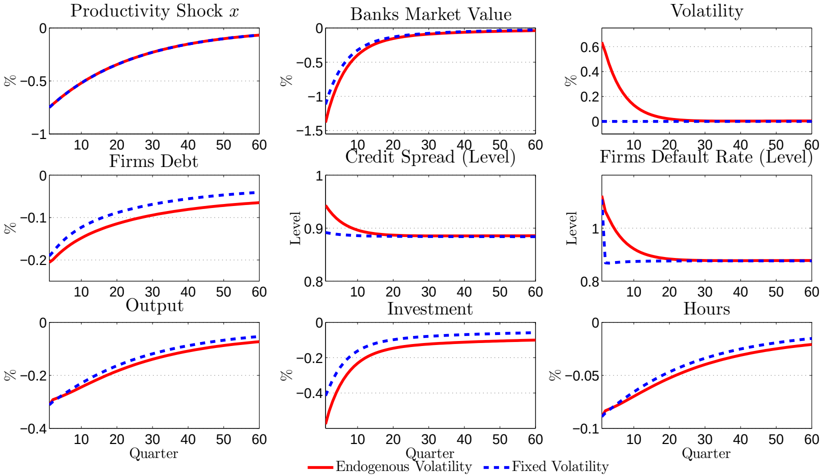


Figure: Model Response to a 1% negative innovation in productivity x

Model with Sticky Wages

Return

- Firms demand a variety of labor for production

$$\Pi(a,k,S) = \max_{\{n_i\}} \left\{ (axk)^{\alpha} \left(\left[\int n_i^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \right)^{1-\alpha} - \int w_i n_i di \right\}$$

- The labor type n_j is monopolistically supplied by a "union"
  - Demand labor from household to produce differentiated labor units.
  - Can reset prices every period with probability $1 - \theta_w$
  - Set wage w_j to maximize expected pay-off

$$\max_{w_j} \mathbb{E}_t \sum_{\tau=0}^{\infty} \theta_w^{\tau} m_{t,t+\tau} \left[w_j - w_{t+\tau}^H \right] n_{j,t+\tau}$$

subject to

$$n_{j,t+\tau} = \left(\frac{w_j}{w_{t+\tau}} \right)^{-\epsilon_w} N_{t+\tau}$$

Model with Working Capital Return

- **Firms** problem is

$$\Pi(a,k,S) = \max_n \left\{ (axk)^\alpha n^{1-\alpha} - (1-\theta^L) w(S) n - \theta^L w(S) n R(S) \right\}$$

where $n = \left(\int n_i^{\frac{\epsilon_W - 1}{\epsilon_W}} di \right)^{\frac{\epsilon_W}{\epsilon_W - 1}}$ and $w(S)n = \int w_i(S) n_i di$.

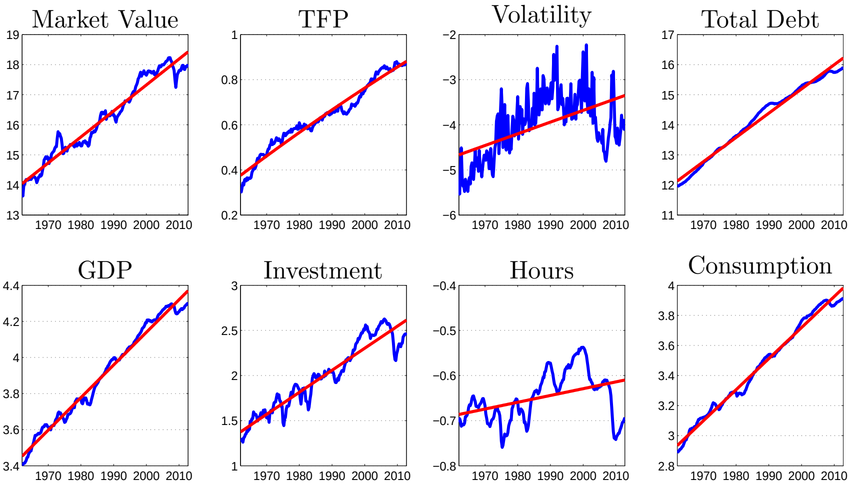
- Interest rate $R(S)$ satisfies

$$1 = R(S)\mathbb{E}_{S'}\left[\tilde{m}(S,S') | S \right]$$

where $\tilde{m}(S,S')$ is **banker's** stochastic discount factor.

2007-2009 Crisis - Data

Return



Note: *Inference for period 1961:1-2012:4. Default rates for corporate non-financial firms with non-investment grade, source Moody's. GDP, Investment and Hours are deflated by working age population. All series normalized to 2007:Q4 = 0. Model with* **flexible wages** *and* **no working capital**.

Model IRF with Sticky Wages and Working Capital

Return

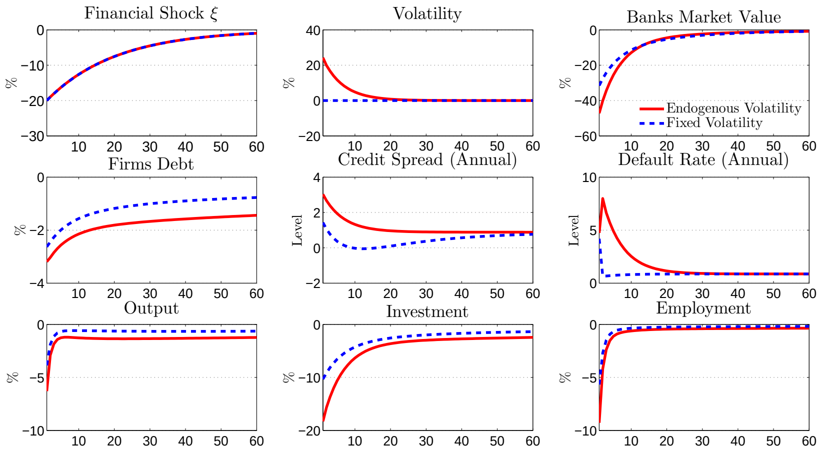
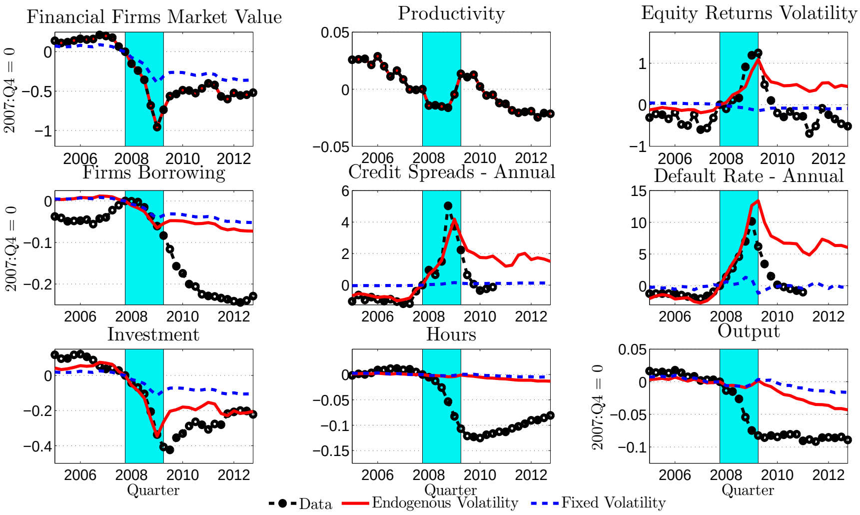


Figure: Model IRF to a 50% decline in Banks Market Value model with **sticky wages** and **working capital**.

2007-2009 Crisis - No Labor Frictions

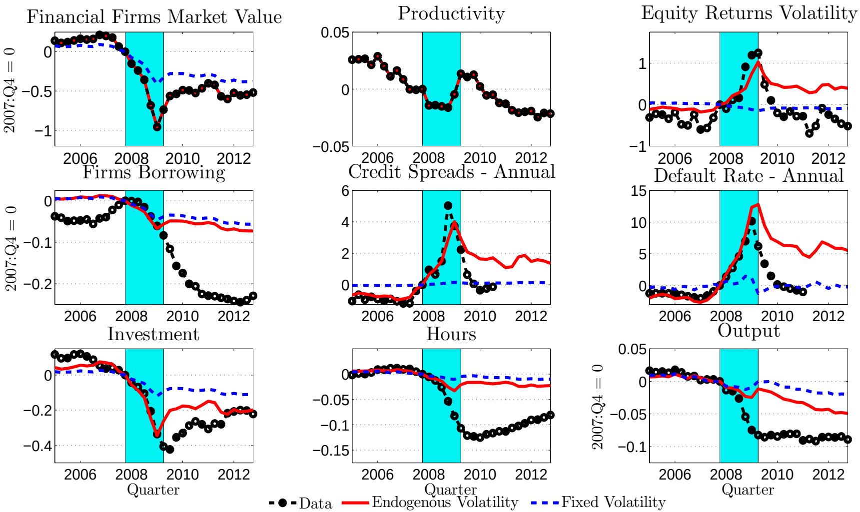
Return · S Wages · W Capital



Note: *Inference for period 1961:1-2012:4. Default rates for corporate non-financial firms with non-investment grade, source Moody's. GDP, Investment and Hours are deflated by working age population. All series normalized to 2007:Q4 = 0. Model with* **flexible wages** *and* **no working capital**.

2007-2009 Crisis - Flexible Wages

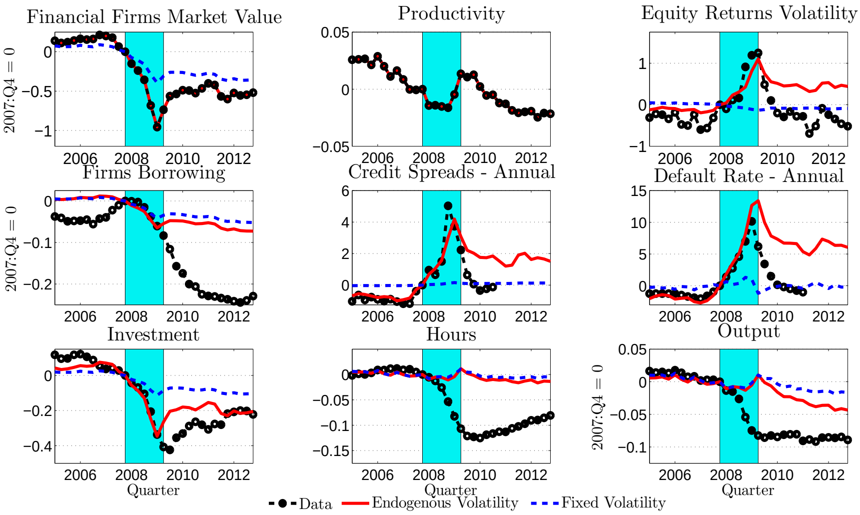
Return



Note: *Inference for period 1961:1-2012:4. Default rates for corporate non-financial firms with non-investment grade, source Moody's. GDP, Investment and Hours are deflated by working age population. All series normalized to 2007:Q4 = 0. Model with* **flexible wages** *and* **working capital**.

2007-2009 Crisis - No working Capital

Return

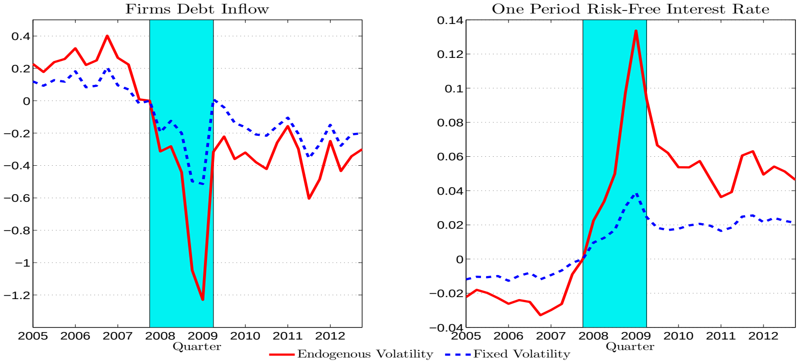


Note: *Inference for period 1961:1-2012:4. Default rates for corporate non-financial firms with non-investment grade, source Moody's. GDP, Investment and Hours are deflated by working age population. All series normalized to 2007:Q4 = 0. Model with* **sticky wages** *and* **no working capital**.

2007-2009 Crisis

Return

How frictions matter? Affects bond prices and the working capital cost.



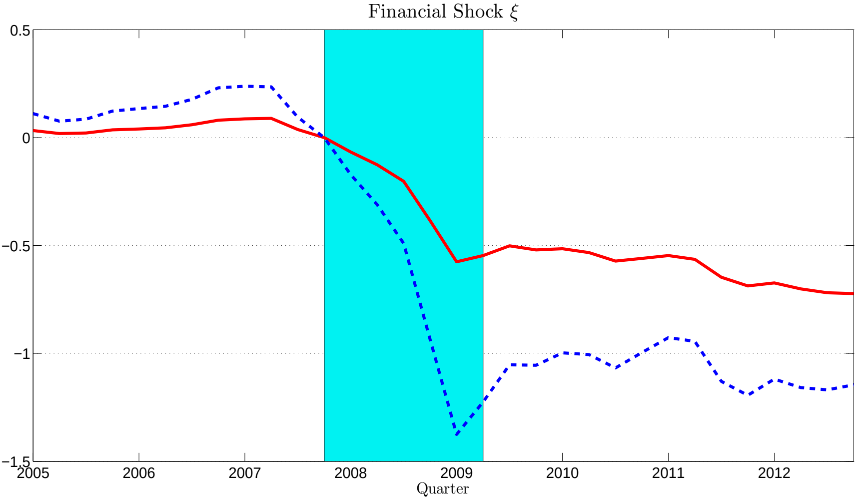
Note: *Inference for period 1961:1-2012:4. All series normalized to 2007:Q4 = 0. Model with* **sticky wages** *and* **working capital**.

Debt inflow $=$ price of debt $\times$ new debt.

Risk-free rate: $1 = R(S)\mathbb{E}_{S'}\left[\tilde{m}(S, S')|S\right]$, with $\tilde{m}(S, S')$ bank's stochastic discount factor.

2007-2009 Crisis - Inferred Shocks

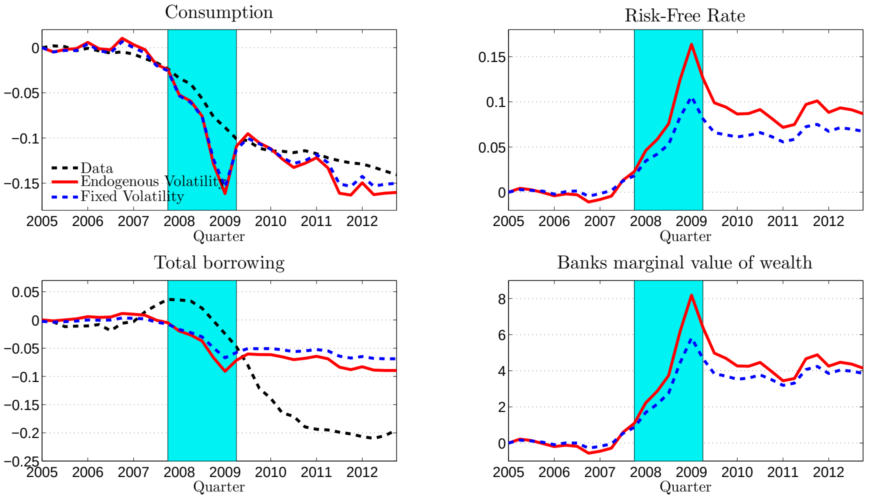
Return



Note: *Inference for period 1961:1-2012:4. Financial shock inferred in each model. All series normalized to 2007:Q4 = 0. Model with* **flexible wages** *and* **working capital**.

2007-2009 Crisis

Return



Note: *Inference for period 1961:1-2012:4. GDP, Investment and Hours are deflated by working age population. All series normalized to 2007:Q4 = 0. Model with* **sticky wages** *and* **working capital**.

Equity Returns Volatility

Return

- Let $r_{i,t-1,t}$ be the return of firm i from period $t-1$ to t

$$r_{i,t-1,t} = \frac{E_{i,t}}{E_{i,t-1} - d_{i,t-1}}$$

- Note that

$$E_{i,t-1} - d_{i,t-1} = \mathbb{E}_{t-1,a}\left[m_{t,t+1} \max\left\{0, E_{i,t}\right\}\right]$$

- From firm's lemma $E_{it}/k_{it} = (1-\tau)\left[a_{i,t}\pi_t - cl_t\right] - \lambda l_t + e_t$.
- Then

$$r_{i,t-1,t} = \frac{(1-\tau)\left[a_{it}\pi_t - cl_t\right] - \lambda l_t + e_t}{\mathbb{E}_{t-1,a}\left[m_{t,t+1} \max\left\{0, (1-\tau)\left[a_{it}\pi_t - cl_t\right] - \lambda l_t + e_t\right\}\right]}$$

- Volatility is

$$Var(r_{i,t-1,t}) = \Upsilon_t^2 Var(a_{it} | a_{it} \geq \underline{\mathbf{a}}_t)$$

where $\Upsilon_t = \frac{(1-\tau)\pi_t}{\mathbb{E}_{t-1}\left[m_{t,t+1} \max\{0,(1-\tau)[a_{it}\pi_t - cl_t] - \lambda l_t + e_t\}\right]}$.

Microevidence

Model testable implication:

- Firms with higher debt issuance should experience lower volatility of returns.

Microevidence

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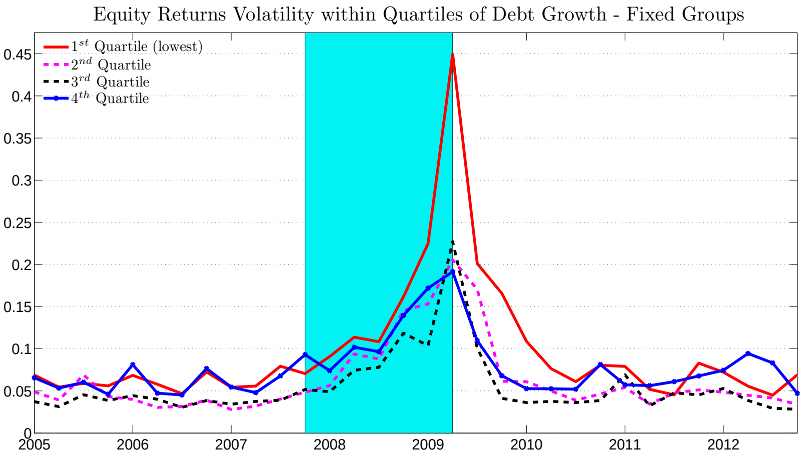
- Firms with higher debt issuance should experience lower volatility of returns.

Exercise: For non-financial firms on Compustat

- Compute **average debt growth rate** during 2007:Q4 to 2009:Q2.
- Divide into quartiles of debt growth rate
  - 1^{st} quartile firms average debt growth rate below 25% percentile
  - 4^{th} quartile firms average debt growth rate above 75% percentile
- Compute dispersion of equity returns **within** quartiles.

Microevidence

Conclusions



Notes: *Quartiles defined by debt growth during the 2007:Q4 to 2009:Q2. Dispersion of returns across firms within each quartile.*

market leverage   Quartile definitions

Microevidence: A panel of firms' volatilities

A volatility measure per firm:

- Let r_{it_d} be the return of **firm** i in **quarter** t in **day** d.
- Volatility measure σ_{it}^2
$$\sigma_{it}^2 = \sum_d (r_{it_d} - \bar{r}_{it})^2$$
with $\bar{r}_{it} = \frac{1}{D} \sum_d r_{it_d}$
- σ_{it}^2 is the average return volatility for **firm** i in **quarter** t.

Microevidence: A panel of firms' volatilities

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- σ_{it}^2 is the average return volatility for **firm** i in **quarter** t.

Exercise: Regression analysis

- Let B_{it} denote firm's i total liabilities.
- Define $\Delta B_{i,t,t+1} = \ln B_{i,t+1} - \ln B_{i,t}$.

Microevidence: A panel of firms' volatilities

Conclusions

- Effect of $\Delta B_{i,t,t+1}$ on $\sigma^2_{i,t+1}$

$$\ln \sigma^2_{i,t+1} = \gamma_t + \chi_i + \beta_{\Delta B} \Delta B_{i,t,t+1} + \beta_X X_{i,t} + \epsilon_{it}$$

Microevidence: A panel of firms' volatilities

Conclusions

- Effect of $\Delta B_{i,t,t+1}$ on $\sigma^2_{i,t+1}$

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Debt Growth	-0.07 [−0.08, −0.07]	-0.10 [−0.11, −0.09]	-0.18 [−0.19, −0.17]	-0.18 [−0.19, −0.17]
ln Assets		-0.19 [−0.20, −0.19]	-0.24 [−0.24, −0.24]	-0.20 [−0.20, −0.20]
ln Market Leverage			0.24 [0.24, 0.24]	0.22 [0.22, 0.21]
ln Profits				-0.03 [−0.03, −0.02]
R^2	13%	38%	38%	34%
obs	659,333	659,329	635,745	430,100

Notes: *Firm's market leverage is the ratio of firm's debt over firm's market value. Firms' debt is total liabilities minus deferred tax liabilities. Firms market value, assets and profits comes from Compustat. Assets corresponds to total assets and profits corresponds to operating income before interest payments and capital depreciation. Returns come from CRSP.*

Microevidence: A panel of firms' volatilities

Conclusions

- Effect of $\Delta B_{i,t,t+1}$ on $\sigma^2_{i,t+1}$

$$\ln \sigma^2_{i,t+1} = \gamma_t + \chi_i + \beta_{\Delta B} \Delta B_{i,t,t+1} + \beta_X X_{i,t} + \epsilon_{it}$$

Debt Growth	−0.07 [−0.08, −0.07]	−0.10 [−0.11, −0.09]	−0.18 [−0.19, −0.17]	−0.18 [−0.19, −0.17]
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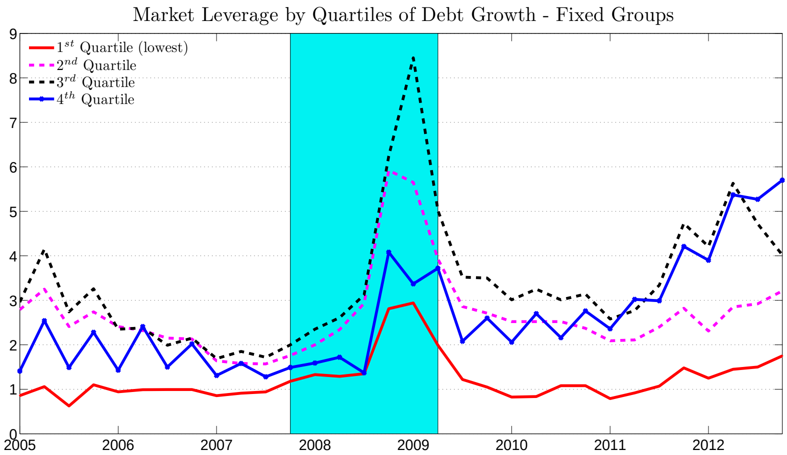
R^2	13%	38%	38%	34%
obs	659,333	659,329	635,745	430,100

Notes: *Firm's market leverage is the ratio of firm's debt over firm's market value. Firms' debt is total liabilities minus deferred tax liabilities. Firms market value, assets and profits comes from Compustat. Assets corresponds to total assets and profits corresponds to operating income before interest payments and capital depreciation. Returns come from CRSP.*

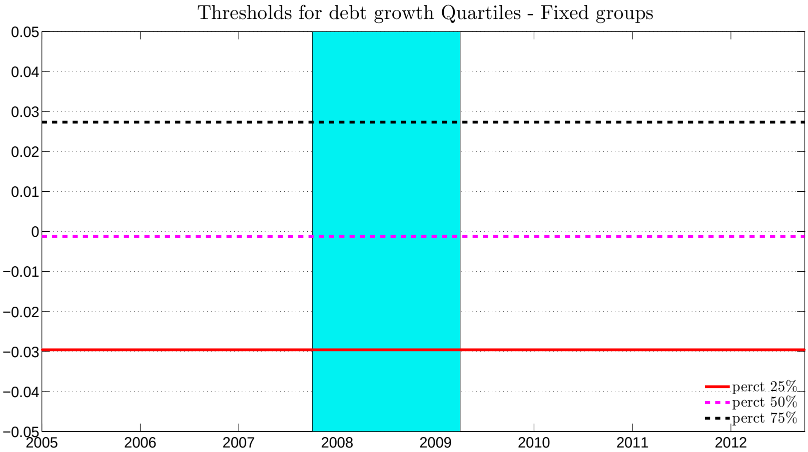
- Find a robust negative effect of debt growth on equity returns volatility.

Microevidence - was not leverage ...



Return

Microevidence - Quartile definitions



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