The Unemployment Accelerator*

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Abstract

What determines a firm's value and the financial conditions it faces? To answer this question, this paper studies the *unemployment accelerator*, a mechanism where workers directly affect the firms' financial conditions, and, in turn, firms' financial conditions feedback again to the real economy. The *unemployment accelerator* builds on two key assumptions: a search friction in the labor market and firms' default risk. The former assumption implies a positive relation between the firm's value and its number of workers; the latter assumption entails a tight connection between the value of the workers and the firm's incentives to default. We develop and estimate a model with these two frictions, together with firm-level heterogeneity and business cycle dynamics, and use to quantify the effect of workers' value on financial conditions. In the context of our model, labor accounts for 32% and 56% of the volatility of default rate and market value respectively, and it is more important than physical capital in this regard. The reason for this quantitative result is that the value of labor is much more volatility than the corresponding one of capital. Finally, we provide compelling micro-evidence of the *unemployment accelerator*: a 10% increase in a firm's number of workers is associated with a 4% increase in its market value and a 6% decline in its probability of default. We show that our model can account for these facts, and that the two key assumptions we make are essential for this.

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1 Introduction

What gives value to a firm? How the firm's value interact with frictional financial markets and the real economy? The macro-finance literature provides an answer built on the *financial accelerator* mechanism: a firm's value determines its financial conditions, and these further feedback on the firm's value and the real economy. Furthermore, because these models are built on a neoclassical set-up, a firm's value is largely driven by the value of capital. Consequently, the quantitative success of these models critically depends on the price of capital being volatile enough. Nevertheless, empirically plausible movements in the price of capital significantly limit these models capacity to propagate business cycle fluctuations.[1]

In order to overcome this limitation, this paper develops the *unemployment accelerator*, a new mechanism where workers act as an asset that directly affects firms' value and financial conditions. Unlike the case of physical capital, we argue that workers are an asset whose value significantly fluctuates over time, thus generating sizable propagation of shocks across firms and over the business cycle. In order to formally establish these ideas, we develop three elements of the *unemployment accelerator*: a micro-foundation, based on a search frictions in the labor market; a quantification of the importance of the workers for firm's financial conditions; and direct micro-evidence in support of the *unemployment accelerator* mechanism.

The *unemployment accelerator* derives from the interplay of two frictions: 1) a search frictions in the labor market; 2) firms' default risk. Because of the search frictions, firms attach a positive value to having a worker since it is costly to hire a new one. Therefore, a firms' market value increases with its number of workers, which directly affects the firm's incentives to default. This implies that workers are qualitatively identical to capital as an asset affecting the firms' value and its financial conditions. Thus, we think of the *unemployment accelerator* as a complement of the well known *financial accelerator*, where firms' financial conditions are mainly driven by the value of physical assets.

To quantify the *unemployment accelerator*, we embed these two frictions in a DSGE model with

[1]For instance, Merz and Yashiv (2007) show that, using realistic movements of the price of capital and reasonable capital adjustment costs, the standard deviation of firms' value is 9 times smaller in a neoclassical model than in the data.

heterogeneous firms and business cycle fluctuations. Firm heterogeneity allows us to test the model with the micro-evidence we provide, as well as to explore the value of workers across the distribution of firms. The business cycle dynamics allow us to study the aggregate propagation implications of the *unemployment accelerator*, and to link the value of workers with (aggregate) labor market conditions. The latter is key to identifyig the size of fluctuations in the value of workers as an asset. Despite its complexity, we use a directed search structure in the labor market (Menzio and Shi, 2010) which renders a highly tractable model and enables us to estimate it.

In the context of our model, we find that the *unemployment accelerator* has a sizable propagation consequences, accounting for more than for 32% and 56% of the firms' aggregate volatility of default rate and market value, respectively. The intuition behind the quantitative success of the mechanism is rather simple: the cost of hiring a worker, and thus the value of a worker, is proportional to the job finding probability, and the empirical standard deviation of the finding probability growth rate is almost fourteen times higher than the one of the price of investment growth rate. Because our model matches these facts, we obtain a substantial contribution of the value of a worker on the aggregate fluctuations in the economy.

At firm-level, the *unemployment accelerator* also generates a significant and persistent relation between a firm's workers and its value. Importantly, we provide evidence of this relation using a panel of US publicly traded firms: in the data, a 10% increase in a firm's workers is associated with a 4% increase in the firm's market value and a 6% decline on its probability to default. As we show, these results are in line with the model predictions, and robust to controls typically used in the literature such as firms' leverage, profitability, or even future firms' performance. Therefore, firm-level data supports the key element of the *unemployment accelerator*, with workers adding value to the firm beyond its capital and profitability. Importantly, because we obtain the same implications in our model, we can provide a structural interpretation to our empirical results.

In order to induce realistic corporate debt moments, we assume firms' debt has long maturity. This debt structure, together with firm heterogeneity and business cycle fluctuations in general equilibrium, make the numerical computation of the model a non-trivial task. To overcome this, we develop an efficient and accurate high-order iterative method to compute the equilibrium policies.[2] As we discuss in section 4.1, this method allows us to solve the model and simulate a panel of firms in just a few seconds. We take advantage of this and estimate the model targeting several firm-level and aggregate moments that are key to the quantitative evaluation of our mechanism.

The *unemployment accelerator* mechanism we develop has important consequences for unemployment, default risk, investment and output, which makes our mechanism important for macroeconomics. At the same time, the *unemployment accelerator* has key implications for job finding and separation rates, which makes the model relevant for labor economics. Finally, because we argue that labor is an asset, our mechanism has implication for firm's valuation (asset pricing) as well as its capital structure decision. This make our new mechanism important for financial economics and corporate finance.

1.1 Related Literature

Our work is related to a stream of recent papers that also discuss the contribution of workers to the value of a firm. The work of Merz and Yashiv (2007) studies the workers' contribution to firms' value in the presence of flexible adjustments cost function, and find that the neoclassical model significantly improves its capacity to account for market value fluctuations once adjustment costs on workers are added.[3] Similarly, the work by Bazdresch, Belo and Lin (2014) provide evidence of a link between employment growth and stock returns predictability using firm-level data, and extend Merz and Yashiv (2007) to explain this link. Similarly, Favilukis, Lin and Zhao (2015) extend previous work to explain how wage rigidity can affect firms' default decisions.

Our work contributes to these papers in two dimensions. First of all, we develop a general equilibrium model that endogenously links the cost of hiring workers to labor market conditions. In turn, the cost—and thus value—of workers is tightly linked to observables that discipline our computations.[4] As we argued above and show below, matching labor market moments is key for our findings. Interestingly, we do this in tractable general equilibrium model with firm heterogeneity. A

[2] See Gomes, Jermann and Schmid (2013) or Miao and Wang (2010) for recent DSGE models analyzing the effect of long-term debt and alternative solution methods.

[3] Related work by these authors are Yashiv (2000) or Yashiv (2016) more recently.

[4] See Merz and Yashiv (2007) who review the literature on hiring costs and argue that is hard to find a consensus.

second important contribution in our paper, is that we extend the analysis on the value of workers to analyze the effect on financial conditions. Thus, we provide a model with a potential to improve the quantitative performance of a large literature on macro-finance models.

The recent papers by Giroud and Mueller (2014) and Chodorow-Reich (2014) provide evidence of a link between employment and the firm's financial conditions. Theoretical models in the same line have recently been developed by Monacelli, Quadrini and Trigari (2011), Buera, Fattal-Jaef and Shin (2013), Arellano, Bai and Kehoe (2011), Petrosky-Nadeau (2011) and Garin (2011) among others. We contribute to these papers by providing new evidence on the relation between workers and financial conditions, as well as a structural model to interpret findings. Importantly, we argue that workers directly affect a firm's financial conditions, a relation not analyzed by these papers.

The rest of the paper is organized as follows. Section 2 presents the model, and Section 3 provides an analytical characterization of its equilibrium. Section 4 discusses the model solution and its estimation. Section 5 discusses the aggregate implication of the *unemployment accelerator*, and Section 6 presents the micro-evidence to support it. Section 7 concludes.

2 Model

In order to study the effects of a firm's worker on its financial conditions, we build a dynamic general equilibrium model with firm heterogeneity and aggregate shocks. We assume decreasing returns to scale technology and persistent idiosyncratic shocks to obtain a realistic firm distribution, which allows us to compare the model with data. We include a default problem at the firm level, which is key to the interaction between the firm's financial decision and the value of its workers. Furthermore, we analyze the effects that this interaction has on labor market dynamics and aggregate outcomes more generally. In order to keep the model tractable, we borrow some modeling assumptions from Lise and Robin (2016), Kaas and Kircher (2015) and Schaal (2015).

2.1 Environment, Preferences and Technology

Time is discrete and indexed by $t \in \{0, 1, 2, \ldots\}$. The economy is populated by a continuum of infinitely-lived workers, lenders and a continuum of firms. The measure of firms and workers are $\tilde{M}$

and $\tilde{N}$, respectively. Firms are divided into incumbents (already operating) or potential entrants.[5] Firms have access to a decreasing returns to scale technology given as

$$y = \eta z k^{\alpha\nu} n^{(1-\alpha)\nu}, \qquad 0 < \nu < 1 \tag{1}$$

where η and z are the aggregate and idiosyncratic productivity shocks respectively, k is the firm's capital and n is its number of workers. Production entails a cost F_k per unit of capital, and a random fixed cost F. We assume that F is *i.i.d.* across firms and time, and distributed as $F \sim G$; and z follows a Markov process with transition matrix $P_z(z, z')$.

In order to hire new workers, firms must post vacancies in a labor market with search frictions (Mortensen and Pissarides, 1994). We assume a directed-search framework: firms can post vacancies in a sub-markets indexed by m, each of which promises a life-time utility m to a worker in case of being hired. Similarly, workers must choose in which sub-market m to search for a job. The tightness of each sub-market is determined in equilibrium, as we discuss below. Firms also invest and accumulate physical capital over time.

To finance its expenditures, firms can issue debt which takes the form of long maturity bonds. In particular, a bond promises to pay a coupon c per unit of outstanding debt every period, and to retire (mature) a fraction λ of the remaining principal.[6] However, since firms may decide to default, debt is risky. Upon default, lenders recover a fraction $R \in [0, 1]$ of the firms capital. Thus, there is an endogenous (default induced) firms exit every period.

Lenders and workers are risk-neutral, with preferences over consumption given as $\mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t C_t]$. We assume lenders are the only agents that buy the firms' debt and they are the ultimately owners of all the firms in the economy. Thus, firms discount cash-flows at the same rate β.

Let $s = (k, n_-, b, z)$ denote the states of the firm, where b and n_- stands for the outstanding debt and workers of firm at the beginning of the period. As we discuss below, firms can hire workers at the beginning of the period before production, and thus the workers n engaged in production

[5]Section 2.6 below discusses firms' entry.

[6]This structure is convenient because of its recursive nature. It have been extensively used recently in the sovereign default literature; see Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012) among others. It has also been recently used in *macro-finance* environments; see Gomes *et al.* (2013) and citations therein. As in all of these papers, we think of $1/\lambda$ as the average maturity of the debt.

– as in equation (1)– may be different than n_{-}. Let Ψ contain all the aggregate shocks in the economy, and let X collect all the aggregate states of the economy – including the aggregate shocks Ψ. Thus X contains all the available information in the economy.

Although the aggregate state X contains all the information in the economy, we will proceed by constructing an equilibrium where individual policies are function of the idiosyncratic state s and the *exogenous* aggregate state Ψ only. As we show below, this is feasible due to the directed search structure assumed and makes the model computations more tractable.

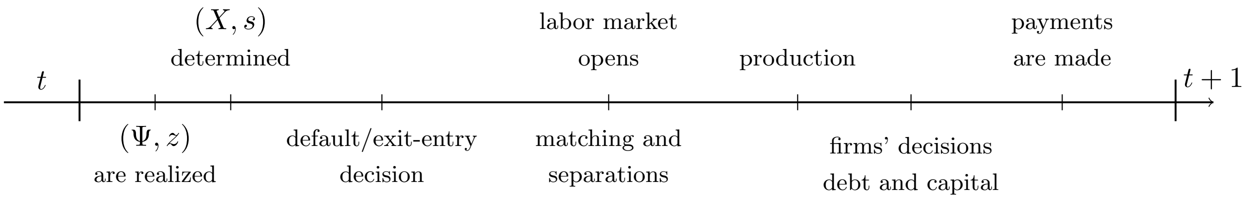

Figure 1: Timing in the model in period t

Figure 1 describes the timing within the period. At the beginning of the period, the idiosyncratic and aggregate shocks (Ψ, z) are realized, and the state (X, s) for each firm is determined. At this point firms decide whether to default or not. Conditional on not defaulting, firms choose a sub-market m where to post vacancies, as well as how many workers to layoff. Simultaneously, unemployed workers select in which sub-market to supply their labor. New hires are engaged in production this period. At this point, firms make their investment and funding (debt and equity) decisions. Simultaneous to these decisions, all payments in the economy are made: firms pay dividends, wages and maturing debt, and benefits are distributed to unemployed workers. The recursive problems of all agents are written just after the firms' default decision.

2.2 Labor Market and the Value of Unemployment

At the beginning of every period, when labor market opens, unemployed workers can select a sub-market m where to search for a job. If he doesn't succeed in finding a job, a worker receives a benefit $\bar{u}(\Psi)$ per period, which can in principle depend on the aggregate exogenous state Ψ.[7]

[7] Allowing the unemployment benefit to depend on the exogenous shocks of the economy would be in line with the evidence in Chodorow-Reich and Karabarbounis (2013). Additionally, it would help us to quantify the macroeconomic

Let $\mathcal{U}(\Psi)$ be the value of a worker who did not find a job this period. Then

$$\mathcal{U}(\Psi) = \max_m \left\{ f(\Psi, m)m + (1 - f(X, m)) \left\{ \bar{u}(\Psi) + \beta \mathbb{E}_{\Psi'} \left[\mathcal{U}(\Psi') \right] \right\} \right\} \tag{2}$$

where $f(\Psi, m)$ is the probability of finding a job in sub-market m when the aggregate state of the economy is Ψ. Notice that (2) incorporates the fact that workers optimally choose the sub-market where to search for a job at the beginning of every period. The unemployed worker choose optimally the trade-off between a higher finding probability and a higher value of a match.

2.3 Labor Contracts and the Value of Employment

We assume that firms and workers can commit to a state-contingent contracts. In particular, a firm offers a contract that specifies four elements: this period wage w, a layoff probability this period when the labor market opens φ, a firm's exit/default decision next period χ, and a continuation value for the worker in case of no default W. Contracts are state-contingent: the default decision as well as continuation values are a function of next period states (Ψ', s', F'). Let

$$\omega = \left\{ w, \varphi, \chi(\Psi', s', F'), W(\Psi', s', F') \right\}$$

denote a contract.

Let $\mathcal{W}(\Psi, s, \omega)$ be the value to a worker of being matched with a non-defaulting firm with state s under a contract ω, when the aggregate exogenous state is Ψ. Then

$$\mathcal{W}(\Psi, s, \omega) = \varphi \left[\bar{u}(\Psi) + \beta \mathbb{E}_{\Psi'} \left[\mathcal{U}(\Psi') | \Psi \right] \right] + (1 - \varphi) \left\{ w + \beta \mathbb{E}_{\Psi', z'} \left[\chi' \mathcal{U}(\Psi') + (1 - \chi') W') | \Psi, z \right] \right\} \tag{3}$$

where $\chi' = \chi(\Psi', s', F')$ and $W' = W(\Psi', s', F')$ are state-contingent decisions, and $\chi' = 1$ means the firm decides to exit. Thus, an employed worker may become unemployed this period because he was laid off (with probability φ), or at the beginning of next period because the firm decided to exit.

effect of increasing the unemployment benefit during a recession.

Notice that workers also commit to the contract ω. In particular, if next period continuation value $W(\Psi', s', F')$ is lower than the unemployment value $\mathcal{U}(\Psi')$, workers stay at the firm.

2.4 Firms

Consider next the problem of an incumbent firm's that decided not to default this period. At the beginning of the period, when the labor market opens, the firm makes its hire and layoff decisions, including in the sub-market where to post vacancies. After his, the firms decide how much to invest in capital, as well as the new debt and equity issuance. Simultaneously, the firm offers contracts $\{\omega_j\}_j$ to all its workers $j \in [0, n]$. These contracts are constrained to deliver at least the life-time utility promised to each worker in the past. We describe next each component of the firm's problem.

If a firm decides to hire $\hat{n}$ new workers, and fire φ of its workers, the number of workers for production this period are

$$n = (1 - \varphi)n_- + \hat{n} \tag{4}$$

where n_- is the number of workers the firm started this period with. Hiring the $\hat{n}$ new workers involves a cost which we describe below. It's worth noting that we impose layoff probabilities φ to be constant across workers. It can be shown that this does not affect the firms' maximal attainable value, and it simplifies exposition.

Denote $\mathcal{P}$ the cash-flow the firm generates from production after paying all its costs. Then

$$\mathcal{P} = \eta z k^{\alpha\nu} n^{(1-\alpha)\nu} - \int_0^n w_j dj - F_k k - F - [(1 - \tau)c + \lambda]\, b \tag{5}$$

The cash-flow of the firm is given by its production, minus three costs: wages $\{w_j\}$ paid to workers, the proportional cost of production F_k, and the cost shock F. Notice that the number of workers comes from equation (5). Now, because the firm only produces if it does not default, it also has to pay its debt: the coupon c and the fraction λ of its outstanding debt b. Notice that there is a tax subsidy τ on coupon payments, which incentivizes the firm to issue debt.[8]

[8]This assumption is typically referred as the *trade-off* theory of firm's capital structure. See Abel (2015) and Graham and Leary (2011) for recent discussions.

Next, let π denote the net-flow the firm obtains from hiring, investment and debt issuance decisions. Then

$$\pi \quad = \quad -\xi i - \frac{c_n}{q(\Psi, m)}\hat{n} + Q(b', k', n, \Psi, z)\hat{b}' - \mathcal{C}^{\varphi}(\varphi, n, n_{-}, \Psi, z) - \mathcal{C}^k(i, k) - \mathcal{C}^b(\hat{b}', b) \tag{6}$$

where i is capital investment, and $\hat{b}'$ is the new debt the firm issues. The variable ξ is the transformation rate from consumption to capital goods, which we model as stochastic and part of the aggregate shocks Ψ. We also allow for adjustment costs $\mathcal{C}^k$ and $\mathcal{C}^b$ for capital and debt respectively.

The cost of posting a vacancy is c_n and $q(X, m)$ is the probability of filling a vacancy if posted in sub-market m. Thus, the cost of hiring $\hat{n}$ new workers in sub-market m is $\frac{c_n}{q(X,m)}\hat{n}$. We also assume a layoff cost function $\mathcal{C}^{\varphi}(\varphi, n, n_{-}, \Psi, z)$, which allows to match a realistic separation rates and labor dynamics.[9] Then, the net resources the firm has is given by $\mathcal{P} + \pi$.

Consider now the contracts the firm is offering. When selecting contracts $\{\omega_j\}$, the firm faces a promise-keeping constraint: if the firm promised in the past a life-time utility $\{W_j\}_{j\in[0,n]}$ to its workers, contracts must satisfy

$$\mathcal{W}(\Psi, s, \omega_j) \geq W_j \qquad \forall j \in [0, n] \tag{7}$$

Thus, past promised utilities $\{W_j\}_{j\in[0,n]}$ is a state to the firm.

The objective of the firm is to maximize the present discount value of its dividend payments d, taking into account that it may optimally decide to exit in the future. Let $\mathcal{J}\left(\Psi, s, F, \{W_j\}_{j\in[0,n]}\right)$ be the value of a firm with state (s, F) that promised life-time utility $\{W_j\}_{j\in[0,n]}$ to its workers,

[9]Absent this cost $\mathcal{C}^{\varphi}(\cdot)$, we would always have corner solutions with either $\varphi = 0$ or $\varphi = 1$.

when the aggregate state of the economy is X. Then

$$\begin{aligned} \mathcal{J}\left(\Psi, s, F, \{W_j\}_{j\in[0,n]}\right) &= \max_{d,i,\hat{b}',\hat{n},m,\{\omega_j\}} \left\{ d + \mathbb{E}_{\Psi',z'}\left[\left\{\mathcal{J}\left(\Psi', s', F', \{W'_{j'}\}_{j\in[0,n']}\right)\right\}^{+} |\Psi, z\right]\right\} \quad (8) \\ \text{subject to} \qquad d &\leq \mathcal{P} + \pi \\ b' &= (1-\lambda)b + \hat{b}', \qquad k' = i + (1-\delta)k \\ W'_{j'} &= \begin{cases} m & \text{if } j' \text{ new worker} \\ W_j(\Psi', s', F') & \text{if } j' \text{ incumbent worker} \end{cases} \\ \text{and} \qquad & (4), \quad (5), \quad (6), \quad (7) \end{aligned}$$

The firm's dividend payments d must be paid with its resources $\mathcal{P} + \pi$. However, a firm can set $d < 0$ which we understand as equity issuance. Notice that the default decision next period χ', included in the contract choice, is incorporated in the term $\{\mathcal{J}\}^{+}$ next period.

Firms find it optimal to issue debt solely because of the tax advantage τ, and would only use dividends/equity absent this advantage. [10] Naturally, once the firm issues debt, it exposes itself to default risk. In turn, the bond price depends on the firm's decisions because these determine the firm's value and thus likelihood to default next period. This is why firms debt and capital choices – b' and k'– appear in the bond price. The firm understands this and internalizes it when making decisions.

The key of our mechanism is that, as a result of the search friction in the labor market, the number of workers increases the firm's value and thus relaxes financial conditions. In this sense, labor works the same way as physical capital.

2.5 Bond Pricing

All of the firms' bonds are purchased by the *lenders*, who have risk-neutral preferences. However, we assume that lenders have a stochastic marginal utility of wealth given by ψ. This is a simplified manner in which we introduce a financial shock that directly affects bond prices.[11]

[10] See Abel (2015) and Graham and Leary (2011) for discussion on the *trade-off* theory.

[11] A stochastic marginal value of wealth can be micro-founded as the results of income shocks on an lender who faces a leverage constraint. See Navarro (2015), who shows this in an environment like Gertler and Kiyotaki (2009).

Accordingly, the bond price is simply the discounted expected value of repayment next period adjusted by the marginal value of wealth today. In particular, for any choice of the firm's debt, capital and workers, the price of the firm bond is

$$\begin{aligned} Q(b', k', n, \Psi, z) &= \psi\beta\mathbb{E}_{\Psi', z', F'}\Big[\left[1-\chi(\Psi', s', F')\right]\left[(c+\lambda)+(1-\lambda)Q\left(\boldsymbol{h}'(\Psi', s', F'), \Psi', z'\right)\right]\Big|\Psi, z\Big] \\ &+ \psi\beta\mathbb{E}_{\Psi', z', F'}\Big[\chi(\Psi', s', F')R\frac{k'}{b'}\Big|\Psi, z\Big] \end{aligned} \tag{9}$$

where $s' = (k', n, b', z')$ and $\boldsymbol{h}'(\Psi, s, F) = \left\{\boldsymbol{b}'(\Psi, s, F), \boldsymbol{k}'(\Psi, s, F), \boldsymbol{n}(\Psi, s, F),\right\}$ collects the firm's optimal policies.[12]

A firm's bond price then has two components. If the firm defaults ($\chi(\Psi', s', F') = 1$), lenders recover a fraction R of the firm's capital pro-rated by the its outstanding debt. If the firm does not default next period, the lender obtains the coupon payment c, the fraction of debt that matures λ, and the market value of the remaining fraction $1-\lambda$. In turn, the market value of debt—$Q(\boldsymbol{h}'(\Psi', s', F'), \Psi', z')$—depends not only on the realization of shocks tomorrow, but also on the policies the firm will follow next period. Thus, lenders correctly forecast firms' policies next period in order to price the debt today.

The presence of future firm's policies in equation (9) makes solving and estimating the model a non-trivial task. In Section 4.1 we discuss an efficient and accurate algorithm to overcome this issue.

2.6 Firms Entry and Free Entry Condition

There is a mass of firms that every period decide whether to enter to the market and start producing or not. Entry is costly and involves paying a start-up fixed cost c_e. Entrants know the aggregate state of the economy Ψ when making the entry decision, and they observe the idiosyncratic productivity z.

We assume that, conditional on no defaulting, a new firm can purchase capital that it uses for

[12] Assuming that household's preferences are risk-neutral is important to maintain the model's tractability. Alternatively, agents would need to forecast consumption each state next period which depends on the firms' distribution over states. To compensate for the lack of a consumption-based asset pricing model, we include the shock ψ. Details on the household and lender problem can be found in Appendix A.

production within same period. Thus, there is no time to build in capital for the entrants.[13] Let $\mathcal{J}^e(\Psi, z)$ be the value of an entrant who did not default. Then

$$\begin{aligned} \mathcal{J}^e(\Psi, z) &= \max_{k_e, b_e, n_e, m, \{\omega_j\}} \left\{ d + \beta \mathbb{E}_{\Psi', z'} \left[\left\{ \mathcal{J}\left(\Psi', s'_e, F', \{W'_{j'}\}_{j\in[0,n']}\right) \right\}^+ |\Psi, z \right] \right\} \quad (10) \\ \text{subject to} \quad d &\leq \mathcal{P} + \pi \\ \mathcal{P} &= \eta z k_e^{\alpha\nu} n_e^{(1-\alpha)\nu} - \int_0^{n_e} w_j dj - F_k k \\ \pi &= -\xi k_e - \frac{c_n}{q(\Psi, m)} n_e + Q(b_e, k_e, n_e, \Psi, z) b_e \\ W'_{j'} &= m \end{aligned}$$

where $s'_e = (k_e, n_e, b_e, z')$. Thus, after having paid the entry cost c_e, the problem of the entrant is very similar to the incumbent one, except that there are no adjustment costs nor time to build in capital.

Free entry condition is given by equalizing the entry cost and the expected value of the firm upon entry, as shown in equation (11). As we argue in Section , this equation will pin down the value of a worker.

$$c_e = \int \{\mathcal{J}^e(\Psi, z)\}^+ \, d\mathcal{H}(z) \quad (11)$$

where $\mathcal{H}$ is the *c.d.f.* of the idiosyncratic productivity for a new firm, and $G(F)$ is the *c.d.f.* of the cost shock.

2.7 Equilibrium Definition

Next we provide next a formal equilibrium definition of the model.

Definition 1 *A recursive equilibrium is given by a set of value functions*

$$\mathcal{U}(\Psi),\ \mathcal{W}(\Psi, s, \omega),\ \mathcal{J}\left(\Psi, s, F, \{W_j\}_{j\in[0,n]}\right), \mathcal{J}^e(\Psi, z)$$

[13] We make this assumption for tractability reasons, so that output for new firms is not zero.

Firms' policy function $\boldsymbol{h}'(\Psi, s, F)$ *and* $s'_e(\Psi, s)$ *for incumbent firms and entrant firms; a bond price function* $Q(b', k', n', \Psi, z)$ *such that:*

- $\mathcal{U}(\cdot)$ *satisfies* (2).
- *Given a bond price function* $Q(\cdot)$*, firm's policies* $\boldsymbol{h}'(\Psi, s, F)$ *solve* (8) *and achieve value* $\mathcal{J}\left(\Psi, s, F, \{W_j\}_{j\in[0,N]}\right)$.
- *Given firm's policies* $\boldsymbol{h}'(\cdot)$*, the price function* $Q(b', k', n, \Psi, z)$ *satisfies* (9).
- *Given a bond price function* $Q(\cdot)$*, entrants policies* $s'_e(\Psi, s)$ *solve* (10) *and achieve value* $\mathcal{J}^e(\Psi, z)$.
- *New firms satisfy the entry condition 11.*
- *Entry is non-negative in any state of the world.*

3 Equilibrium Characterization

The model presented in the previous section can be solved in a simple manner. In particular, in spite of the flexibility of arrangements between firms and workers, we show that a surplus maximization problem can be used to obtain the firm's optimal policies. The advantage of doing so is that the heterogeneity of promised values to workers does not affect the surplus, and thus the dimensionsionality of the problem is drastically reduced. Normatively, this result implies that the firm's incentives are aligned with the value of the match.

3.1 A Surplus Maximization Problem: an Equivalence Result

Let $\mathcal{S}(\Psi, s, F)$ be the physical surplus obtained by a firm and its workers. In particular

$$
\begin{aligned}
\mathcal{S}(\Psi, s, F) &= \max_{d,i,\hat{b}',\hat{n},m',\varphi,\chi'} \left\{ d + \beta \mathbb{E}_{\Psi',z'} \left[\left\{ \mathcal{S}(\Psi', s', F') \right\}^{+} | \Psi, z \right] \right\} && (12) \\
\text{subject to} \quad d &\leq \mathcal{P} + \pi \\
\mathcal{P} &= \eta z k^{\alpha\nu} n^{(1-\alpha)\nu} - \hat{u}(\Psi) n - F_k k - F - \left[(1-\tau)c + \lambda \right] b \\
\pi &= -\xi i - \left[\left(\frac{c_n}{q(\Psi, m)} + (m - \mathcal{U}(\Psi)) \right) \hat{n} \right] + Q(b', k', n, \Psi, z) \hat{b}' && (13) \\
&- \mathcal{C}^{\varphi}(\varphi, n, n_-, \Psi, z) - \mathcal{C}^{k}(i, k) - \mathcal{C}^{b}(\hat{b}', b) \\
d &\leq \mathcal{P} + \pi \\
n &= (1-\varphi) n_- + \hat{n} \\
k' &= i + (1-\delta) k, \qquad b' = (1-\lambda) b + \hat{b}'
\end{aligned}
$$

where $\hat{u}(\Psi) = \mathcal{U}(\Psi) - \beta \mathbb{E}_{\Psi'} \left[\mathcal{U}(\Psi') | \Psi \right]$.

The surplus maximization problem in (12) is very similar to the firm's problem in (8) except for two key differences. First of all, the distribution of promised values $\{W_j\}$ does not matter for the surplus, since it is a transfer between the firm and its workers. For the same reason, wages do not affect surplus, but only the value that unemployment value $\hat{u}(X)$ to workers forgone by being matched with a firm. Accordingly, the promise keeping constraint (7) does not affect the surplus. Consequently, the controls in the surplus problem are not distributional objects any more.

Notice that, from the point of view of the surplus, the cost of hiring a new worker does not only incorporate the vacancy cost c_n, but also the life-time utility that the new worker gains for begin in the match, this is $m - \mathcal{U}(X)$. Interestingly, the sub-market m where new hires are made, only affect the surplus through the cost of hiring this period, but doesn't affect the continuation value of the surplus since it's a transfer between firms and workers. Thus, it's optimal to hire in the sub-market with lowest cost. Denote this cost $\kappa(\Psi)$, which is given as

$$
\kappa(\Psi) = \min_{m} \left\{ \frac{c_n}{q(\Psi, m)} + (m - \mathcal{U}(\Psi)) \right\} \tag{14}
$$

We show below how $\kappa(\Psi)$ is determined by the free-entry condition, as well as how it determines the value of a worker for a firm.

The next proposition formalizes the equivalence between the firm's problem and the surplus maximization. It is very similar to the results and derivations in Kaas and Kircher (2015) and Schaal (2015).

Proposition 1 (Equivalence Result) *The firm's problem in (8) and the surplus maximization problem in (12) are equivalent in the following sense*

(i) The surplus and the firm's value satisfy

$$\mathcal{S}(\Psi, s, F) = \mathcal{J}\left(\Psi, s, F, \{W_j\}_{j\in[0,n]}\right) + \int_0^n (W_j - \mathcal{U}(\Psi))\, dj$$

(ii) If a policy $\left\{i, \hat{b}', \hat{n}, m, \varphi, \chi'\right\}$ *maximizes the surplus, then there exist wages* $\{w_j\}$ *and continuation values* $\{W_j'\}$ *such that a contract* $\omega_j = \left\{w_j, \varphi, \chi', W_j'\right\}$ *is consistent with the promise keeping constraint (7), and* $\left\{i, \hat{b}', \hat{n}, m, \{\omega_j\}\right\}$ *solves the firm's problem and achieves value* $\mathcal{J}\left(X, s, F, \{W_j\}_{j\in[0,x]}\right)$.

(iii) Conversely, if a policy $\left\{i, \hat{b}', \hat{n}, m, \{\omega_j\}\right\}$ *maximizes the firm's problem; then* $\left\{i, \hat{b}', \hat{n}, m, \varphi, \chi'\right\}$ *solves the surplus' problem and achieve* $\mathcal{S}(\Psi, s, F)$.

Proposition 1 is rather intuitive. Because there are no contractual frictions between firms and workers, it is optimal to maximize the value of the match and then somehow distribute it. As long as the joint surplus is positive, there is always a contract sophisticated enough so that the match can continue, both parts benefit and past promises are kept. Thus, the incentives of the firms are aligned with those of the match, and their policies are identical. Appendix C contains the proof of this and the next propositions.

We can use the simpler surplus maximization problem to characterize some of the firm's policies. In particular, we show that most of the firm's policies are independent of the cost shock F, except for default, which follows a threshold policy: firms with a cost shock F high enough will find it optimal to default and exit the economy.

Proposition 2 (Firm's policies) *A firm finds it optimal to default if and only if its cost shock is above a threshold* $\underline{\boldsymbol{F}}(X,s)$ *given as*

$$\underline{\boldsymbol{F}}(\Psi,s) = \eta z k^{\alpha\nu} n^{(1-\alpha)\nu} - \hat{u}(\Psi)n - F_k k - [(1-\tau)c + \lambda]\, b + v(\Psi,s) \tag{15}$$

where $v(\Psi,s) = \mathcal{S}(\Psi,s,F) - \mathcal{P}$. *Then, a firm with state* (Ψ,s) *defaults with probability* $1 - G\left(\underline{\boldsymbol{F}}(\Psi,s)\right)$.

If a firm does not default, its policies for investment, vacancy posting and debt issuance are independent of the cost shock. Denote $\boldsymbol{k}'(\Psi,s)$, $\boldsymbol{b}'(\Psi,s)$ *and* $\boldsymbol{n}(\Psi,s)$ *the resulting states for capital, debt and workers next period, respectively.*

Proposition 2 tightly characterizes the firm's default decision. The more productive the firm is, or the more promising the continuation value $v(\cdot)$ is, the higher the default threshold and the less likely the firm is to default. Analogously, the larger the firm's debt the more likely it is to default. At the same time, the cost shock does not affect the firm's policies because it is *i.i.d.* and does not contain any information about the future.

As a corollary of Proposition 2, we can use the default decision derived in equation (15) to provide a tighter expression for the bond price in equation (9).

Corollary 1 (Firm's policies) *For any choice of next period debt, capital and workers, the price of the firm bond is given as*

$$\begin{aligned} Q(b',k',n,\Psi,z) \quad &= \quad \psi\beta\mathbb{E}_{\Psi',z'}\Bigg[G\left(\underline{\boldsymbol{F}}(\Psi',s')\right)\left[c+\lambda+(1-\lambda)Q\left(\boldsymbol{b}'(\Psi',s'),\boldsymbol{k}'(\Psi',s'),\boldsymbol{n}(\Psi',s'),\Psi',z'\right)\right] \\ &+ \quad \left[1-G\left(\underline{\boldsymbol{F}}(\Psi',s')\right)\right]R\frac{k'}{b'}\Bigg|\Psi,z\Bigg] \end{aligned} \tag{16}$$

where $\underline{\boldsymbol{F}}(X',s')$ *is the default threshold in* (15)

3.2 Firms' Marginal Value of a Worker

We can now formally establish the marginal value of a worker for a firm. In particular, we show next that a firm's value increases with its number of workers, while its probability to default decreases, and the price of its debt increases. Although is not crucial for the results, it is simpler to consider the case with short-term debt ($\lambda = 1$ and $c = 0$) and a layoffs adjustment cost that is independent of workers.[14]

Following proposition formally characterizes the amrginal value of a worker.

Proposition 3 (Marginal Value of a Worker) *If it is costly to hire a new worker ($\kappa(\Psi) > 0$) the marginal value of a worker is positive and given as*

$$\frac{\partial \mathcal{S}(\Psi, s, F)}{\partial n_-} = (1 - \varphi(\Psi, s))\kappa(\Psi) > 0 \tag{17}$$

Furthermore, the probability of no defaulting is proportional to the marginal value of a worker, and thus decreases in the number of workers

$$\frac{\partial G(\Psi, s)}{\partial n_-} = g(\Psi, s)\frac{\partial \mathcal{S}(\Psi, s, F)}{\partial n_-} > 0 \tag{18}$$

where $G(\Psi, s) = G(\underline{\boldsymbol{F}}(\Psi, s))$ and $g(\Psi, s) = g(\underline{\boldsymbol{F}}(\Psi, s))$ Finally, for the case of short term debt ($\lambda = 1$ and $c = 0$), the bond price increases in the number of workers

$$\frac{\partial Q(b', k', n, \Psi, z)}{\partial n} = \psi\beta\mathbb{E}_{\Psi', z'}\left[\frac{1}{g(\Psi', s')}\frac{\partial \mathcal{S}(\Psi', s', F')}{\partial n}\right] > 0 \tag{19}$$

Finally, if contact rate functions are given as $\tilde{f}(\theta) = \theta(1 + \theta^\gamma)^{-1/\gamma}$ and $\tilde{q}(\theta) = \frac{\tilde{f}(\theta)}{\theta}$ - with θ being the v-u ratio, the cost of hiring $\kappa(\Psi)$ is proportional to the job finding probability and given as

$$\kappa(\Psi) = c_n(1 - f(\Psi)^\gamma)^{-\frac{1+\gamma}{\gamma}}$$

Proposition 3 is rather intuitive and the key intuition in the model. In particular, as long as it's costly to hire a new worker, firms will attach value to ones the already have. Reasonably, firms will hire worker until its marginal value equalizes the cost of hiring a new worker, precisely as equation (17) shows. Consequently, a firm with more workers will have a higher value and thus

[14] Both of these assumptions could be relaxed and results would remain unchanged. We choose this special for simplicity of exposition.

less incentives to default, and a higher market price for its debt. This is what equations (18) and (19) show. Thus workers are an asset to the firm, in the same way that capital is.

3.3 Free Entry Condition

Using Proposition 1, we can use the surplus value to characterize the entry problem of the firm in equation (10) as follows

$$\begin{aligned} \mathcal{J}^e(\Psi, z, F) &= \max_{k_e, b_e, n_e} \left\{ d + \beta \mathbb{E}_{\Psi', z'} \left[\left\{ \mathcal{S}\left(\Psi', s'_e, F'\right) \right\}^+ | \Psi, z \right] \right\} \\ \text{subject to} \quad d &\leq \mathcal{P} + \pi \\ \mathcal{P} &= \eta z k_e^{\alpha\nu} n_e^{(1-\alpha)\nu} - \hat{u}(\Psi) n_e - F_k k - F \\ \pi &= -\xi k_e - \kappa(\Psi) n_e + Q(b_e, k_e, n_e, \Psi, z) b_e \end{aligned} \tag{20}$$

where $s'_e = (k_e, n_e, b_e, z')$, and we already substituted the optimal choice of m by

$$\kappa(\Psi) = \min_m \left\{ \frac{c_n}{q(\Psi, m)} + (m - \mathcal{U}(\Psi)) \right\}$$

The advantage of expressing $\mathcal{J}^e(\cdot)$ as in equation (20) is that it does not involve choosing a continuum of continuation values – as it's the case in (10).

Finally, $\kappa(\Psi)$ is pined down by the free-entry condition (11), which we repeat below for convenience

$$c_e = \int \left\{ \mathcal{J}^e(\Psi, z, F) \right\}^+ d\mathcal{H}(z) dG(F) \tag{21}$$

where $\mathcal{H}$ is the *c.d.f.* of the idiosyncratic productivity for a new firm, and $G(F)$ is the *c.d.f.* of the cost shock.

3.4 A Model without the Unemployment Accelerator

To highlight the role of labor as an asset in our model, we compare our search friction environment with a frictionless labor market model. In order to do this in a tractable manner, we construct a partial equilibrium model where only a continuum of firms are present.

More precisely, we assume that the firm problem remains the same as before (see equation (8)), with two differences. First of all, firms do not need to post vacancies and can costlessly hire new workers. This is equivalent to setting the cost of posting vacancies to zero: $c_n = 0$. The second key difference is that the firm takes wages as given, which we assume to be a constant $\bar{w}$, and there is no contracting problem between firms and workers. We also use the bond pricing equation as in (9).

We solve and estimate the frictionless labor market model using the same techniques we use for our benchmark model. We also target the same set of moments. Appendix D contains a detailed description of the frictionless model.

4 Heterogeneous Firms Model Estimation

This section describes how we solve and estimate the model. We use an accurate algorithm to efficiently solve our heterogeneous firms model, suitable to handle some complications that arise in the presence of long-term debt. We estimate most of the model parameter using a moment matching estimation procedure as proposed by Chernozhukov and Hong (2003), which allows us to make robust inferences about the predictions of the model. We target several firm-level statistics, which allows us to take the rich cross-sectional implications of our model to the data. We also calibrate some parameters that have common values in the literature.

4.1 Solution Strategy

The model structure makes its solution very simple: we first obtain the firm's policies from the surplus maximization problem in (12), then we solve the entry problem in (20), and then compute all aggregate quantities. Because the surplus maximization is a common problem, we can use standard perturbation techniques to efficiently and accurately describe its solution. However, as recently pointed out by Gomes *et al.* (2013), an issue arises with perturbation techniques in the presence of long-term debt. We briefly explain this problem next, as well as the solution algorithm we use to overcome this issue.

Because the firm has the monopoly of its debt, it internalizes the effect that its debt choice, as well as capital and labor choice, has on its bond price. This is captured, for instance, by the derivative of the price function $\frac{\partial Q(b',k',n,\Psi,z)}{\partial b'}$. In particular, letting $\boldsymbol{h}'(\Psi,s) = \{\boldsymbol{b}'(\Psi,s), \boldsymbol{k}'(\Psi,s), \boldsymbol{n}(\Psi,s)\}$ collect the firm's policies, we can use equation (16) to compute the price derivative as

$$\begin{aligned}\frac{\partial Q(b',k',n,\Psi,z)}{\partial b'} &= \beta\psi\mathbb{E}_{\Psi',z'}\left[g(\underline{\boldsymbol{F}}(\Psi',s'))\frac{\partial\underline{\boldsymbol{F}}(\Psi',s')}{\partial b'}\left[(c+\lambda)+(1-\lambda)Q\left(\boldsymbol{h}'(\Psi',s'),\Psi',z'\right)\right]|\Psi,z\right] \\ &+ \beta\psi\mathbb{E}_{\Psi',z'}\left[G(\underline{\boldsymbol{F}}(\Psi',s'))\sum_{\mathcal{Y}=\{b,k,n\}}\frac{Q_{\mathcal{Y}}(\boldsymbol{h}(\Psi',s'),\Psi',z')}{\partial\mathcal{Y}}\frac{\partial\boldsymbol{\mathcal{Y}}'(\Psi',s')}{\partial b'}|\Psi,z\right] \\ &+ \beta\psi\mathbb{E}_{\Psi',z'}\left[g(\underline{\boldsymbol{F}}(\Psi',s'))\frac{\partial\underline{\boldsymbol{F}}(X',s')}{\partial b'}\theta\frac{k'}{b'^2}|\Psi,z\right] \end{aligned} \tag{22}$$

where $s' = (b',k',n,z')$ and $\frac{\partial\boldsymbol{\mathcal{Y}}'(\Psi',s')}{\partial b'}$ is the derivative—next period—of the firm's policy function with respect to today's debt choice b'.

As equation (22) shows, we need information about the firm's policies derivatives $\frac{\partial\boldsymbol{\mathcal{Y}}'(X',s')}{\partial b'}$ in order to compute an equilibrium.[15] However, equilibrium conditions do not provide any additional information about these derivatives, and it can be shown that further differentiating equilibrium conditions do not help since one always ends up with one more unknown than equations. Importantly, without having the policies's derivatives, we cannot solve for the non-stochastic steady-state of the model, which complicates computing the solution using standard perturbation techniques.

We overcome this problem by using an iterative procedure of local approximation to the model solution. In particular, for any policy $\boldsymbol{\mathcal{Y}}$, we compute a second-order approximation around the non-stochastic steady-state of the model policies as

$$\ln\boldsymbol{\mathcal{Y}}'(\mathcal{Z}) \approx \phi_0^{\mathcal{Y}} + \phi_1^{\mathcal{Y}}\ln\mathcal{Z} + (\ln\mathcal{Z})'\phi_2^{\mathcal{Y}}(\ln\mathcal{Z}) \qquad \forall\boldsymbol{\mathcal{Y}} = \boldsymbol{b}',\boldsymbol{k}',\boldsymbol{n}' \tag{23}$$

where $\mathcal{Z} = [\Psi,s]'$ collects the aggregate exogenous and firm's idiosyncratic states. Let $\phi^{\mathcal{Y}}$ collect all the coefficients in (23), and notice that this is all the information we need to compute the policy function derivatives. The iterative procedure we use is as follows: for a given guess $\phi^{\mathcal{Y}}$, we compute

[15] Analogous equations for the price derivatives with respect to capital and labor, $\frac{\partial Q(b',k',n,X,z)}{\partial k'}$ and $\frac{\partial Q(b',k',n,X,z)}{\partial n}$, are also part of the equilibrium conditions.

the non-stochastic steady state of the model and the second order approximation of the policies around this point; this delivers an implied set of coefficients in equation (23), call it $\hat{\phi}^{\mathcal{Y}}$; we iterate until $\phi^{\mathcal{Y}} \approx \hat{\phi}^{\mathcal{Y}}$. Details on this procedure can be found in Appendix A.1.

We use a second-order approximation because, as we show below, firms' idiosyncratic shocks are too volatile to be accurately described by a first-order approximation. However, higher order perturbation techniques often result in explosive paths and it is not obvious how to check for stability conditions. Thus, we follow the pruning technique for non-linear systems proposed by Andreasen, Fernandez-Villaverde and Rubio-Ramirez (2016), which includes second order terms in the solution and simultaneously allow us to check for stability conditions in the model. We provide more details in Appendix A.1.

4.2 Data Description

We use annual firm-level balance sheet data to estimate key model parameters. We also use quarterly aggregate data from different sources. We describe next the data we use, and more details can be found in Appendix B.

Firm-level data mostly comes from Compustat-Fundamentals Annual, for the period 1983 to 2015. Table 1 describes all the variables that we used for the computation of the micro-moments in the GMM estimation. We use PPENTQ (net property plant and equipment) as the measure of the firm's physical capital in the model; we construct firm's debt as the sum of DLCQ (debt in current liabilities), DLTTQ (long-term debt total), minus CHEQ (cash and short-term investments); we use CAPXY (capital expenditures) minus SSPE (sales of property) as firm's investment; and SALEQ (sales) as the output produced by the firm. For the change in debt, we used DLTIS (long-term debt issuance) minus DLTR (long-term debt reduction) plus the difference between USTDNC (debt in current liabilities net change) minus CHECH (cash and term investment net change). For firms' number of workers, we use EMP (employees).[16]

We omit firms with SIC between 6000 and 6999 (financial firms) or between 4900 and 4999

[16] The variable EMP is a "Miscellaneous Items" and not available at quarterly frequency. Although it is understood that employment is reported with measurement error in COMPUSTAT, the recent work by Davis, Haltiwanger, Jarmin and Miranda (2006) report a correlation of 0.89 with establishment level data (Longitudinal Database), although the correlation is weaker for smaller firms.

(regulated firms), and drop firms with less than 5 years or with negative assets, capital or sales. The key firm-level variables for the estimation are the growth rate of debt, capital and employment. For capital, we compute the growth rate, for firm i in period t, as $\Delta k_{it} = i_{it}/0.5 * (k_{it-1} + k_{it})$, where i_{it} is investment and k_{it} is capital.[17] Similarly, we compute employment and debt growth as $(n_{it} - n_{it-1})/0.5*(n_{it-1} + n_{it})$ and $(b_{it} - b_{it-1})/0.5*(b_{it-1} + b_{it})$ respectively. For the computation of each moment, we drop 1% outliers and de-mean each growth rate with the mean across time and across each firm. Appendix B.1 describes the details of the moments' computations and table 2 describes the estimated micro-moments.

We use quarterly aggregate data from different sources. We use BLS for labor market variables, and NIPA and FRED for standard macroeconomic variables like output, investment and nominal interest rates. We obtain total debt for non-financial business from Flow of Funds, partly because we found discrepancies in business cycle properties between this measure and the result of aggregating firms' debt from Compustat. This discrepancy is not present for investment or employment—see tables 2 and 4. We use the time series of credit spread constructed by Gilchrist and Zakrajsek (2012). Finally, we use the time series computed by Schmitt-Grohé and Uribe (2011) for the price of investment. See Table 3 for a description and sources of each variable, and Section B.4 for the computation of each variable in the data.

4.3 Estimation: A Simulated Method of Moments Approach

We estimate most of the model parameters using a Simulated Method of Moments (SMM): that is, we select parameters in order to minimize the distance between data moments and the same moments generated by the model. Following the recent work by Lise (2012), we use a Laplace Type estimator as introduced Chernozhukov and Hong (2003).[18] This procedure constructs a pseudo-likelihood criterion function in the form of a quasi-posterior density over the model parameters. We favor this approach, instead of a standard extremum estimator, because it is computationally attractive and less likely to get stuck at local minimums. It simultaneously allows us to make inference

[17] See Section 4.2 for data description of each variable and Appendix B for more details.

[18] Recent papers using the same estimation approach include Jarosch (2015), Lamadon (2014) and Lise, Meghir and Robin (2016).

(i.e. compute standard errors) about the parameters estimates together with the model implied moments observed in data. We implement the estimation by using a random walk Metropolis-Hasting algorithm and uniform priors.

4.3.1 Model Estimation: Firm-Level Statistics and Macroeconomic Aggregates

We exploit the firms' heterogeneity generated by the model in our estimation strategy. In particular, we target several cross-sectional moments of firm-level variables, which allows us to make the model consistent with micro-statistics. We also target aggregate moments commonly used in the literature. We explain next the moments we target and the parameters we estimate.

We target the cross-sectional standard deviation and persistence of debt, capital, output and employment growth rates. In particular, for a given quarter t, we compute the standard deviation of Δk_{it} across firms i, and target the average over time t as the standard deviation measure. Similarly, for a given quarter t, we compute the correlation of Δk_{it} and Δk_{it-1} across firms i, and target the average over time t as the capital growth persistence measure. Analogous calculations are done for employment, output and debt growth. Importantly, we perform the exact same computations in the data as we do in our model-simulated panel.

We also target the cross-sectional contemporaneous correlation of capital, output and employment growth with debt growth.[19] We do this because this micro-statistic contains relevant information about the magnitude of financial frictions in the economy. However, we found the volatility of debt growth to be too high to be matched by our model using plausible parameters. Thus, we added a measurement error to debt growth, to account for additional sources of debt issuance not incorporated in the model.[20]

The aggregate moments we target are the standard deviation, persistence and correlation with output of investment, employment, credit spreads, debt, finding rates, separation rates and output. We filter the data, as well as the model generated data, using a Hodrick-Prescott filter with a smooth parameter of 1600. Additionally, we target several mean values including mean investment

[19] That is, the correlation across firms i of debt growth ΔB_{it} with capital and employment growth Δk_{it} and Δn_{it}, respectively.

[20] See Strebulaev and Yang (2013) for a review of empirical puzzles about the way firms make leverage decisions.

rate, credit spread, recovery rates, default rates and unemployment rates.

We use these targets to estimate many of the model parameters. We assume quadratic adjustment costs in capital, employment and debt as $\mathcal{C}^k K(i,k) = \frac{\phi_k}{2}\left(\frac{i-\delta_k k}{k}\right)^2 k$, $\mathcal{C}^\varphi(\varphi, n', n, \Psi, z) = \frac{\phi_n}{2}\left(\frac{n'}{n}-1\right)^2 n - \bar{\varphi}\frac{\varphi^2}{2}n$ and $\mathcal{C}^b(\hat{b}', b) = \frac{\phi_b}{2}\left(\frac{\hat{b}'-\lambda b}{b}\right)^2 b$ respectively, and estimate $\{\phi_K,\ \phi_N,\ \phi_B\}$. We assume the unemployment benefit is a constant $\bar{u}(\Psi) = \bar{u}$ and estimate $\bar{u}$. We assume that the idiosyncratic productivity shock z follows an AR(1) in logs and estimate its persistence and variance $\{\rho_z, \sigma_z\}$. We also estimate the tax benefit τ, the recovery rate upon default R, the fixed cost of production per unit of capital F_K.

Because we use a second-order approximation, just a few moments of the idiosyncratic shock distribution $G(\cdot)$ affect equilibrium conditions. In particular, only four moments, all evaluated at the default threshold in the non-stochastic steady-state, show up in the second-order approximation: these are the level (*c.d.f.*) of the distribution, its slope (*p.d.f.*), and the first and second derivatives of the *p.d.f.*. Denote these four values as G, g, g_1 and g_2. Because the cost shock also implies a resource cost for those firms that do not default, the average value of the cost shock for non-defaulting firms also shows up in the equilibrium conditions. Denote the steady-state value of this cost shock as as F^e. We estimate these five values related to the idiosyncratic cost shock distribution $\{G, g, g_1, g_2, F^e\}$ since this is all we need to know about the idiosyncratic cost shock distribution $G(\cdot)$ in order to compute the second approximation of the model.

The remaining parameters of the model are calibrated using common values in the literature. We set the discount factor to $\beta = 0.98$, the span of decreasing returns to $\nu = 0.85$, and the exponent of capital in the production function to $\alpha = 0.4$. We assume a capital depreciation rate of $\delta_k = 0.025$. Regarding debt institutional parameters, we assume an average maturity of debt of six years and set $\lambda = 1/24$, and normalize the coupon to $c = 0.0101$ so that the price of a risk-free bond is one in steady state. We estimate the cost of posting vacancies c_n and the cost of entry c_e. We assume the matching function given by $f(\theta) = \theta(1+\theta^\gamma)^{1/\gamma}$ with $\gamma = 1.6$. We assume an AR(1) process for the log of aggregate productivity η and price of investment ξ_t, with persistence and innovation variance for productivity of $\{\rho_\eta, \sigma_\eta\} = \{0.85, 0.008\}$ in line with the Solow residual and with persistence and innovation variance for the price of investment of $\{\rho_\xi, \sigma_\xi\} = \{0.88, 0.0033\}$

in line with the price of investment in the data. Finally, we also assume an AR(1) in logs for the financial shock ψ with zero variance in the innovation.[21] Table 1 shows the calibrated parameters.

Table 1 shows the 17 total parameters we estimate.

Table 1: Models Parameters

Calibrated Parameters	Value	Target/Source
Preferences and Technology		
Discounting (β)	0.98	Standard
Span of Control (ν)	0.85	Standard
Capital elasticity (α)	0.4	Standard
Capital depreciation (δ_K)	0.025	Standard
Matching elasticity (γ)	1.6	Schaal (2015)
Exogenous separation ($\overline{\varphi}$)	0.033	Shimer (2005)
Debt Contract Calibrated Parameters		
Debt maturity (λ)	0.0417	Gomes *et al.* (2013)
Debt coupon (c)	0.0101	Normalization
Exogenous Aggregate Shocks		
Productivity (ρ_η, σ_η)	$(0.85, 0.008)$	Solow residual process
price of investment (ρ_ξ, σ_ξ)	$(0.88, 0.0033)$	Schmitt-Grohé and Uribe (2011)
Risk-Premium shock (ρ_ψ, σ_ψ)	$(0.88, 0.00)$	-
Estimated Parameter	**Prior**	***Mean***
Unemp. benefit ($\bar{u}$)	$\mathcal{U}\,[0.1, 0.9]$	0.22
Cost vacancies (c_n)	$\mathcal{U}\,[2, 10]$	7.33
Cost entry (c_e)	$\mathcal{U}\,[2, 10]$	1.19
Tax benefit (τ)	$\mathcal{U}\,[0.01, 0.4]$	0.23
Recovery rate (R)	$\mathcal{U}\,[0.2, 0.6]$	0.17
Adjustment Costs		
Capital (ϕ_k)	$\mathcal{U}\,[0.01, 6]$	1.11
Employment (ϕ_n)	$\mathcal{U}\,[0.01, 6]$	0.99
Debt (ϕ_b)	$\mathcal{U}\,[0.01, 6]$	0.28
Firm-Level TFP and Cost Shocks		
TFP persistence (ρ_z)	$\mathcal{U}\,[0.4, 0.99]$	0.86
TFP variance (σ_z)	$\mathcal{U}\,[0.05, 0.14]$	0.036
c.d.f. level (G)	$\mathcal{U}\,[0.997, 0.998]$	0.9974
p.d.f. level (g)	$\mathcal{U}\,[0.0005, 0.02]$	0.0085
p.d.f. first derivative (g_1)	$\mathcal{U}\,[-0.02, 0.005]$	-0.0089
p.d.f. second derivative (g_2)	$\mathcal{U}\,[-0.02, 0.005]$	-0.010
average cost (F^e)	$\mathcal{U}\,[-0.03, 0.5]$	0.0047

[21] See appendix B.4 for the construction of each variable.

4.3.2 Model Fit: Micro Heterogeneity and Aggregate Implications

The model has a good fit with firm-level cross-sectional moments, as Table 2 shows. It generates a standard deviation of output and employment growth in line with the data, as well as the comparatively lower volatility of capital growth. Similarly, the model generates a lower persistence in debt and employment growth, together with intermediate persistence in output growth and higher persistence in capital growth.

The model can also account for the large volatility of debt growth, although a fraction of this is due to the assumed measurement error in the estimation. In any case, a good match is also obtained with many other moments related to debt growth, which implies that the model performs well in explaining in explaining moments of debt growth as well. Overall, the model fits firm-level cross-sectional moments well.

Firm-level cross-sectional moments are important since it imposes discipline to the adjustment cost in capital and labor; thus the volatility of the marginal value of capital and workers coming from the adjustment costs.

Regarding aggregate moments, the model can match business cycle moments of employment, investment and output, although it falls short of generating enough volatility for the labor market variables and the financial variables. Also, the model over-estimate the volatility of aggregate capital/investment. Thus, we will over-estimate the contribution of capital with respect to the volatility of financial conditions.

Notice that the model with walrasian labor market has a better fit with aggregate variable since the model exhibits fixed wages (in our model the opportunity cost of wages mitigates the effect of the aggregate shocks). But even if aggregate variables are more volatile in the model with walrasian market, the financial variables are less volatile, since the unemployment accelerator is absent in this model.

Table 2: Moments Data-Model

	Data	Model UA	Model no UA	Data	Model UA	Model no UA
Firm-level moments						
	Growth rates std			Growth rates persistence		
debt	0.373	0.373	0.373	−0.044	0.003	0.005
employment	0.148	0.185	0.127	0.054	0.113	0.229
output	0.200	0.152	0.141	0.12	0.267	0.220
capital	0.096	0.101	0.057	0.51	0.508	0.505
Aggregate moments						
	Mean			Std (times 100)		
Output	–	–	–	1.625	1.344	1.833
Investment (rate)	0.143	0.164	0.192	7.421	10.250	7.406
Finding rate	0.425	0.426	–	8.981	3.572	–
Credit Spread	0.414	0.58	0.708	23.889	2.177	1.460
Debt (leverage)	0.660	0.899	0.609	2.458	1.278	0.378
Default rate	0.250	0.287	0.451			
	Persistence			Correlation with Output		
Output	0.850	0.824	0.837	–	–	–
Investment	0.796	0.621	0.610	0.838	0.566	0.692
Finding rate	0.525	0.663	0.927	0.790	0.871	–
Credit spreads	0.810	0.736	0.735	−0.405	−0.896	−0.737
Debt	0.925	0.963	0.958	0.316	0.321	0.246

5 Macro Effects of the *Unemployment Accelerator*

5.1 The Mechanism Behind the *Unemployment Accelerator*

In this section we compute impulse response functions to gain intuition of the workings of the model. In particular, we compute the model response to the aggregate shocks η

An decrease in aggregate productivity η induces an immediate reduction in firm's output, as Figure 2 shows. At the same time, the firm decreases the number of vacancies and hire fewer workers, as well as cuts down investment. The lower productivity, togethwe with fewer workers and capital, makes the firm less valuable and default rate increases. Consequently, the average price of debt declines and firms' issue less debt.

The reduction in firms' value makes entry less attractive, what directly reduces the cost of hiring κ. The reduction in the cost of hiring has two effects: first, it incentives hiring because of its lower cost. Second, it reduces the value of the current stock of workers at the firm; thus increasing the firms' incentives to default. This second effect reduces even more the firms' value with the further feedback on the real economy.

5.2 Labor and Capital Contribution to Financial Conditions

In this section, we quantify the effect of the *unemployment accelerator* across firms and over the business cycle. In particular, we compute the fraction of the volatility in fluctuations of default rate and market value that are attributed to the value of a worker. As we show below, this contribution is large in our benchmark model, but zero without the search friction.

Recall that default probability for a firm with state s, when the aggregate (exogenous) state o the economy is Ψ, is given y $G(\Psi, s)$, where $s = (k, n_{-}, b, z)$. Then, the change in default rate for firm i in period t can be approximated as

$$\Delta G_{it} = G_{k,it}\Delta k_{it} + G_{n,it}\Delta n_{it} + G_{b,it}\Delta b_{it} + G_{z,it}\Delta z_{it} + G_{\Psi,it}\Delta\Psi_t \tag{24}$$

where G_y is the derivative of $G(s, \Psi)$ with respect to $y = k, n, b, z, \Psi$, and notice that these vary both across firms and time. Then, a contribution of labor and capital to the overall firm default

Figure 2: Firm's Impulse Response to Aggregate TFP shock η

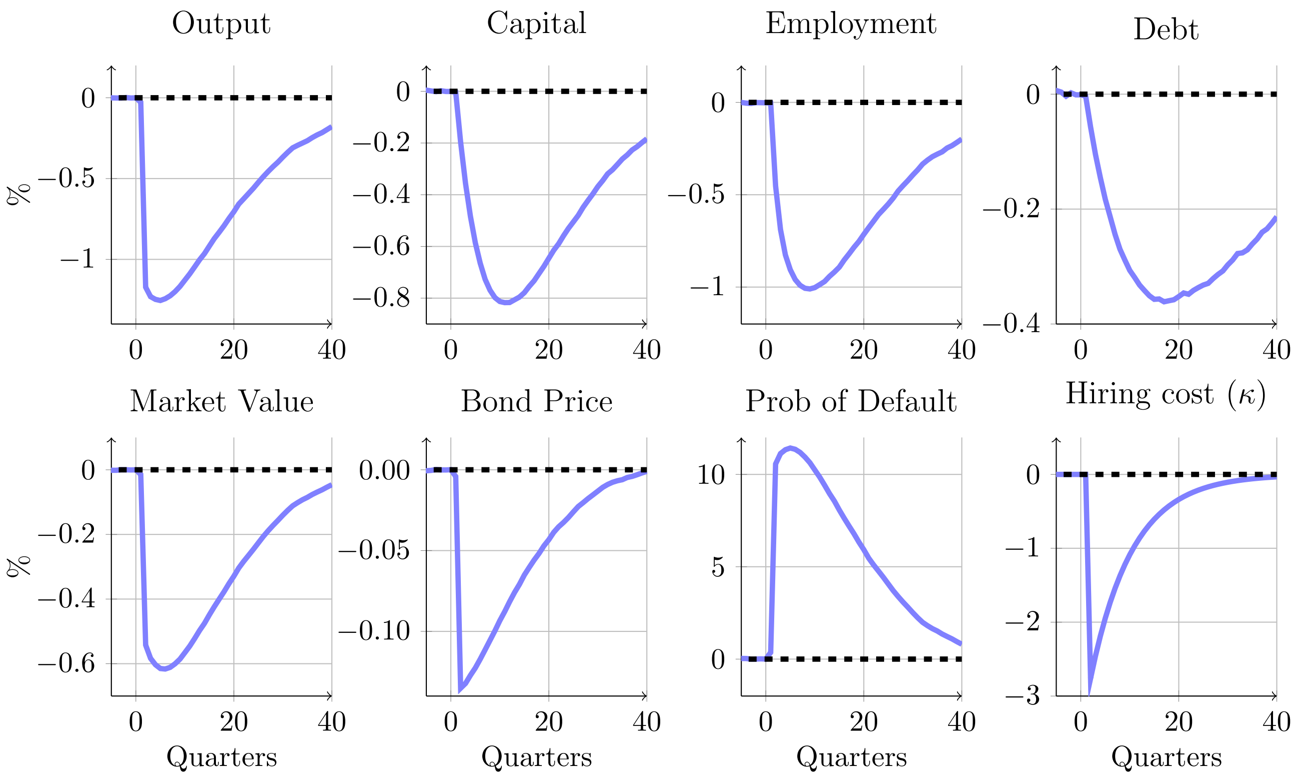

Table 3: "Variance Decomposition" at Idiosyncratic and Aggregate Level

	Contribution with *UA*			Contribution with no *UA*		
	Labor	Capital	Productivity	Labor	Capital	Productivity
Variance Decomposition at Idiosyncratic Level						
Default rate	27%	14%	57%	0%	27%	97%
Market value	33%	14%	52%	0%	17%	97%
Variance Decomposition at Aggregate Level						
Default rate	32%	24%	0%	0.2%	40%	0%
Market value	56%	42%	0%	0%	46%	0%

This table describe the variance decomposition of credit spread at idiosyncratic and aggregate following the definitions 25 and 27 probability is given as

$$\text{labor contribution} = \frac{\mathbb{V}(G_{n,it}\Delta n_{it})}{\mathbb{V}(\Delta G_{it})},$$
$$\text{capital contribution} = \frac{\mathbb{V}(G_{k,it}\Delta k_{it})}{\mathbb{V}(\Delta G_{it})},$$
$$\text{productivity contribution} = \frac{\mathbb{V}(G_{z,it}\Delta z_{it})}{\mathbb{V}(\Delta G_{it})}, \tag{25}$$

where the variances on equations (25) are taken over firms i and periods t.

The first two rows of table 3 describe the contribution coming from labor and capital. As we can see the labor contribution is around 30% for default rate and market value. These magnitudes are twice as much as the capital contribution. The contribution of the workers value in the model without the *unemployment accelerator* is zero for labor and increases to 27% for capital. The first two rows of table 3 describe the total variance coming from both idiosyncratic and aggregate shocks. Since idiosyncratic shocks are ten times larger than aggregate shocks, this decomposition reflects largely the effect of idiosyncratic shocks.

Similarly as before, the aggregate change in default rate can be approximated as

$$\sum_i \Delta G_{it} = \sum_i G_{k,it}\Delta k_{it} + \sum_i G_{n,it}\Delta n_{it} + \sum_i G_{b,it}\Delta b_{it} + \sum_i G_{z,it}\Delta z_{it} + \sum_i G_{\Psi,it}\Delta \Psi_t \tag{26}$$

Then, an aggregate quantification of labor and capital to overall default rate is given as

$$\text{labor contribution} = \frac{\mathbb{V}(\sum_i G_{n,it}\Delta n_{it})}{\mathbb{V}(\sum_i \Delta G_{it})}, \qquad \text{capital contribution} = \frac{\mathbb{V}(\sum_i G_{k,it}\Delta k_{it})}{\mathbb{V}(\sum_i \Delta G_{it})} \tag{27}$$

The last two rows of table 3 describe the contribution coming from labor and capital. As we can see the labor contribution drops relative to capital with respect to the previous exercise, but it is still bigger than the capital contribution.

6 Micro-Evidence of the *Unemployment Accelerator*

In this section, we argue that a firm's number of workers reduces its probability to default. We show that our model is consistent with these findings: labor is an asset to the firm, and thus increases its value and decreases its probability to default. Importantly, we show that, absent search frictions in the labor market (No *UA* model), the model cannot account for these facts, and thus conclude that the search friction is crucial.

Empirically establishing the marginal contribution that workers have on a firm's probability to default is not an easy task. The reason is that, arguably, a firm's number of workers correlates with several characteristics of the firm: for instance, a very productive firm is likely to be more valuable, with more assets and workers. Furthermore, there is also no obvious instrument that is highly correlated with the number of a firm's workers but not with firm's other characteristics.

In order to overcome this omitted variable issue, we proceed by adding several controls that can provide information about the firm's probability to default. After controlling for all these variables, we interpret the remaining correlation as the *marginal contribution* that the number of workers have on the firm's probability to default.

When selecting controls, we include the state variables suggested by our model, as well as many other variables that our model misses but could account for the firm's value. This includes news about future performance of the firm; lagged values of the firm's profits; and measure of the firm's asset composition. We also allow the marginal effect of workers to vary with the firm's size. Our findings are robust to all of these controls: a firm's number of workers has a positive *marginal* contribution to its market value, and a negative *marginal* contribution to the firm's probability to default.

We start by discussing the evidence on the marginal effect of workers on the firms' probability to default by only taking into account the model implied state variables. We use this as a benchmark specification to compare our model with.

6.1 Worker Effect on Probability of Default

To understand the effect of employment on the firm's probability to default, we first compute the effect that variables *other than employment* have on firms' value. We then evaluate how responsive the residual component is to the firm's number of workers. In order to do this, we first briefly describe a measure of firm's probability of default in the data, and then move to compare model and data.

6.1.1 A Measure of Default Risk

While in the model we have a clear measure of default risk, this is not the case in the data. To overcome this, we follow the seminar work by Merton (1974), based on the Black-Scholes-Merton option-pricing model, to construct a *distance to default* measure for all the firms in our panel. The algorithm consists of two steps: first, inferring the firm's assets value given its market value; and second, using an option-price formula to compute the firm's probability of default. Because our computations closely follow the ones in Duffie (2011) and Gilchrist and Zakrajsek (2012), we keep exposition brief.[22]

Let A_{id} be the total assets value of the firm i on day d, and assume that A_{id} follows a geometric Brownian motion with instantaneous drift μ_{id}^{A} and volatility σ_{id}^{A}. It is also assumed that the firm defaults if its value A_{id} is below a accounting-liability based measure L_{id}, at some point between date t and $t+T$.[23] The insight of Merton (1974) is that the firm's equity value V_{id} can be viewed as an call-option on the underlying value of the firm's assets A_{id}, with a strike price L_{id}, and maturity date $t+T$. Then, the value of equity V_{it} is give by the Black-Scholes-Merton option-pricing formula as follows

$$V_{id} = A_{id}\Phi(d_{id}^{1}) - e^{-r_t T} L_{it}\Phi(d_{id}^{2}) \tag{28}$$

[22] See Appendix B.3 for more details.

[23] Black-Scholes-Merton original formulation actually assumes that L_{id}, μ_{id}^{A} and σ_{id}^{A} are constants, while we allow for time variation as in Bharath and Shumway (2008).

where r_t denotes the instantaneous risk-free rate, and d^1_{id} and d^2_{id} are given as

$$d^1_{id} = \frac{\log(A_{id}/L_{id}) + (r_t + 0.5\sigma^2_{id})T}{\sigma^2_{id}T} \qquad d^2_{id} = d^1_{id} - \sigma_{id}\sqrt{T} \tag{29}$$

Operationally, we proceed as follows. The equity value V_{id} is constructed using daily CRSP data on stock prices. The default threshold L_{id} is computed as the firm's short-term debt plus one-half of her long-term debt. The source of liabilities data is Compustat at quarterly frequency, and the data is linearly interpolated to obtain daily observations. The risk-free interest rate r_t is assumed to be 10-year US Treasury yields..[24] Finally, to obtain $\{\mu^A_{id}, \sigma^A_{id}\}$, we follow an iterative procedure: given a guess for A_{id}, we compute $\{\mu^A_{id}, \sigma^A_{id}\}$ from a 250-day rolling window, and then use equation (28) to obtain a new guess of A_{id}. We iterate until convergence.

Finally, we compute two variables: the distance to default DF_{id}, and the probability to default Φ_{id}

$$\begin{aligned} DF_{id} &= \frac{\log(A_{id}/L_{id}) + \mu_{id}T - 0.5\sigma^2_{id}T}{\sigma_{id}\sqrt{T}} \\ \Phi_{id} &= \Phi\left(-DF_{id}\right) \end{aligned}$$

Quarterly observations for each variable are taken as averages: $DF_{it} = \frac{1}{D_t}\sum_d DF_{id}$ and $\Phi_{it} = \frac{1}{D_t}\sum_d \Phi_{id}$ for all quarters t.

6.1.2 Evaluating the Effect

To evaluate the effect of a worker on the firm's probability to default, we proceed as follows: first, we compute the residual of the firm's probability to default after controlling for several firm variables; second, we evaluate how this residual depends on the firm's number of workers. In particular, for both data and model, we estimate the following equation

$$\ln \Phi_{it} = \alpha^{\Phi}_i + \gamma^{\Phi}_t + \beta^{\Phi} X_{it} + \varepsilon^{\Phi}_{it} \tag{30}$$

[24] See Appendix B.3 for details.

where X_{it} includes the same controls as before. Then, we regress the residual ε_{it}^{Φ} on the one of employment ε_{it}^{N} as computed in (32). In particular

$$\varepsilon_{i,t+h}^{\Phi} = \delta_0 + \delta_h^{\Phi,N} \varepsilon_{it}^{N} + u_{i,t+h} \tag{31}$$

where Φ_{it} is firm's i probability to default measure in quarter t, and X_{it} is a set of firm-specific controls. We select the set of controls guided by our theoretical model described in Section 2. These controls include the firm's capital, liabilities, the profits-to-assets ratio and the investment-to-assets ratio.[25] While total assets and liabilities are meant to capture firms' financial conditions, the inclusion of profits and investment reflects firms' performance (a proxy for productivity). Importantly, other than employment, these variables are enough statistics in our model to describe the firm's problem. We also allow for firms' fixed effects (α_i^{Φ}) to represent systematic idiosyncratic differences, as well as time dummies (γ_t^{Φ}) to capture aggregate macroeconomic conditions.

Equation (30) has the typical variables that affects firms value in most *macro-finance models*. Thus, we think of the residuals ε_{it}^{Φ} as the component of the firm's default risk that cannot be accounted for by standard determinants in the literature. We are interested in understanding how these residuals depend on firms' employment. To this end, we estimate an analogous equation for employment. In particular, for both data and model, we estimate

$$\ln N_{it} \quad = \quad \alpha_i^N + \gamma_t^N + \beta^N X_{it} + \varepsilon_{it}^N \tag{32}$$

As before, equation (32) regresses employment on typically used controls for the firm X_{it}. Thus, we think of a positive ε_{it}^{N} as a firm with employment higher than expected in that quarter.

If employment is not a significant determinant of the firm's default risk, other than because of its correlation with variables X_{it}, the residuals ε_{it}^{Φ} and ε_{it}^{N} should be uncorrelated.[26] We show next that this is not the case.

A firm with employment higher than expected, as captured by a positive ε_{it}^{N}, induces a persistent

[25] See Section 4.2 and Appendix B for data details.

[26] No correlation in the residuals is equivalent to including workers in equation (30) and obtaining a coefficient of zero. This would be the case if X_{it} contains all relevant information to determine a firm's value.

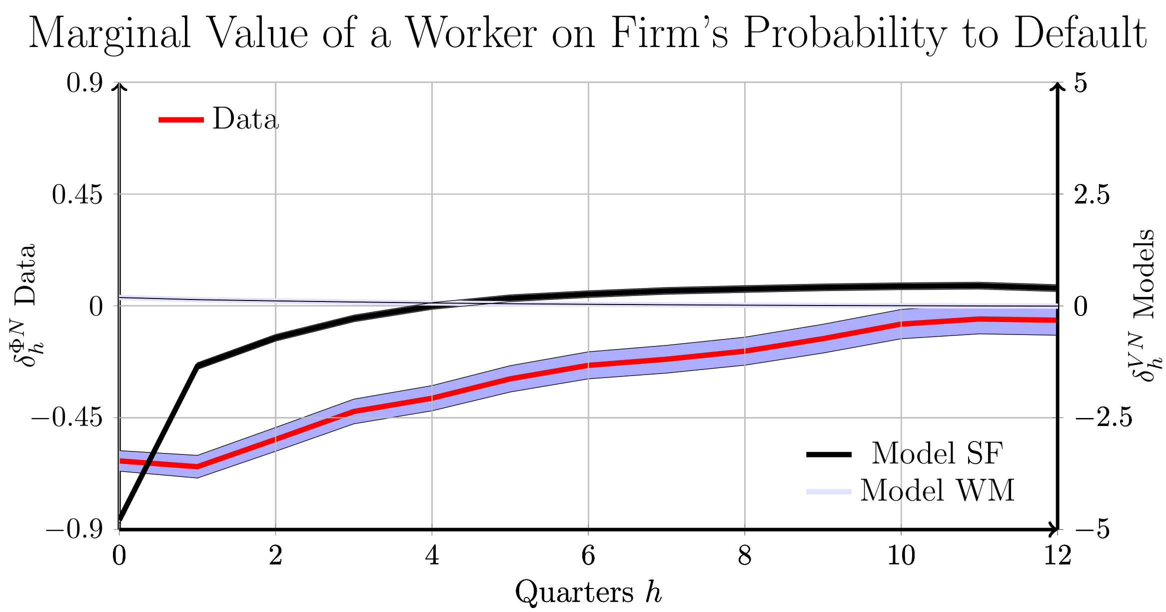

Figure 3: Marginal Value of a Worker on Firm's Probability to Default

increase in its market value. Figure 3 shows this by plotting, for different horizons h, the estimate δ_h of the following regression using the data panel

$$\varepsilon^{\Phi}_{i,t+h} = \delta_0 + \delta_h^{\Phi,N} \varepsilon^N_{it} + u_{i,t+h} \tag{33}$$

In order to test our model, we replicate the same set of regressions (30), (32) and (33) using a model-generated panel of firms, both for the *unemployment accelerator* model as well as with its variation without the *unemployment accelerator*. Figure 3 shows that the *unemployment accelerator* model can replicate the negative empirical relation between number of workers and firm's probability to default. However, the model without the search friction cannot account for this. We conclude that the search friction is essential for accounting for our empirical findings.

Although the *unemployment accelerator* model has the qualitatively correct implication, it overstates the marginal effect that workers have on the firm's probability to default. In the data, a 10% increase in the number of workers induces a 6% decrease in the firm's probability to default, while the decline is 60% in the model with less persistence.

Our model guided the selection of controls X_{it} used in equations (33) and (33). Naturally, the model abstracts from several features that, if present in data, could bias our results. In the online appendix E we show that our results are robust to controlling for a large number of misspecification: a firm's number of workers significantly increase as its market value and decreases its probability to default.

7 Conclusions

The reliance of macro-finance model in the value capital implies a lack of propagation in these models, since neither the price or quantity of capital can generate enough fluctuation of financial conditions. To overcome this feature, we develop the unemployment accelerator where the workers are an asset to the firm affecting the firms value and financial conditions. With the context of our model, we found a quantitative significant contribution in the value of worker in the financial conditions, even more than capital.

Going forward, we see two important dimension to explore with respect to the interaction of the labor and financial market. The first dimension consist in friction in the labor market and how they affect the financial conditions in the unemployment accelerator. The second dimension is related with firms value at the firm-level and aggregate level: what dimensions with respect to the labor force affect the firm's rate of return and firm's value? For example, the composition between skill and unskilled labor could the affect the stochastic process of the firm's value; or maybe the labor force turnover.

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A Lender Problem

There is a continuum of investors with linear preferences over consumption with a discount factor given by β. Let $V(\Lambda, \omega)$ be the value of an investor with wealth ω and aggregate state Λ. Then

$$\mathcal{I}(X,\omega) = \max_{\{C,b'(h,z)\}} \left\{C + \beta \mathbb{E}_{X',F'}\left[\mathcal{I}\left(X',\omega'\right)\middle| X\right]\right\} \tag{A.34}$$

subject to

$$\begin{aligned}
\omega &\geq C + \frac{1}{\psi)}\int_{\tilde{s}} Q(h',X,z)b'(\tilde{s})d\tilde{s} - \int d(s)d\mu(s) \\
\omega' &= \int_{\tilde{s},s'} (1-\chi(\Psi',s',F'))(c+\lambda+(1-\lambda)Q(h'(X,\cdot),X',z'))b'(\tilde{s})d\mu(s'|\tilde{s})d\tilde{s} \\
&+ \int_{\tilde{s},s'} \chi(X',s',F')\left(Rk'\frac{b'(\tilde{s})}{b'}\right)d\mu(s'|\tilde{s})d\tilde{s}
\end{aligned}$$

where $h' = (b', k', n)$ is the period choice of debt, capital, and labor; $\mu(s'|\tilde{s})$ is the conditional distribution of the firm's state next period conditional on $\tilde{s}$. The first line is the investor budget constraint. The total income is given by firms' profit and initial wealth. The total expenditure is given by consumption plus the investment in the corporate bonds. Notice that the asset structure of this economy is extremely large. In particular, the investor is choosing to invest its wealth from a continuum of bonds indexed by (h', z)—firms' idiosyncratic state. The demand for bonds is denoted by $b'(h', z)$. For no defaulting firms, each bond has a period payment equal to a coupon plus the maturity of the bond; plus the continuation value of the non-maturing bonds given by the re-selling price. For the defaulting firms, each investor keep a fraction $\theta\frac{b'(h',z)}{b'}$ of the capital, where the former ratio represent the ratio debt hold by the investor with respect to the total.

Online Appendix: Not for Publication

The Unemployment Accelerator

Andres Blanco **Gaston Navarro**

A Model: Computation Details

A.1 Model Solution: Iterative Procedure

In this section we describe the iterative method to compute the second order approximation of the firms policy function. Let $h, h_Y \in R^{6\times 6}$ for $Y \in \{K, N, B\}$ and let

$$x_t = [k_{t-1,t}, n_{t-1,t}, b_{t-1,t}, z_t, \eta_t, \psi_t, \xi_t] \in R^{6\times 1} \tag{A.1}$$

When we apply perturbation methods we log-linearize the state variables and linearize the control variables

- Step 1: Guess $h_0 \in R^{6\times 6}$ in the set of diagonal matrices with positive diagonal.
- Step 2: Given h_i, compute the policy using first order perturbation methods on the equilibrium conditions. In this step $\phi_1^{\mathcal{Y}}$ is obtain from h_k . Denote h_{i+1} the solution of the equilibrium equations that satisfies

$$x_t = h_{i+1}x_{t-1} + \Sigma\epsilon_t \tag{A.2}$$

- Step 3: If $\frac{norm(h_{i+1}-h_i)}{1+norm(h_i)} < \epsilon$, then go to step 4— where norm denotes the euclidian norm of the vectorize matrix. If not, go to step 2.
- Step 4: Given h_i, compute the equilibrium solution using second order perturbation methods. Let $\phi_2^{\mathcal{Y}}$ be the quadratic form for the quadratic term in the policy of $\mathcal{Y} \in \{k, n, b\}$.
- Step 5: Given $h_i, h_{ki}, h_{ni}, h_{bi}$, compute the policy using second order perturbation methods on the equilibrium conditions. In this step $\phi_1^{\mathcal{Y}}$ is obtain from h_k and $\phi_2^{\mathcal{Y}}$ are obtain from $h_{\mathcal{Y}}$.Denote $h_{i+1}, h_{\mathcal{Y}i+1}$ the solution of the equilibrium equations that satisfies

$$Y_t = h_{i+1}^{Y}Y_{t-1} + x_{t-1}^{T}h_{Yi+1}^{i}x_{t-1} + \Sigma\epsilon_t \tag{A.3}$$

  where $\mathcal{Y} \in \{k, n, b\}$ and $h_{y+1}^{\mathcal{Y}}$ is the row of the matrix corresponding to $\mathcal{Y}$.
- Step 6: if $\frac{norm(h_{i+1}-h_i)+\sum_Y norm(h_{Yi+1}-h_{Yi})}{1+norm(h_i)} \geq \epsilon$, go to step 5. If not, you find the equilibrium policy using second order perturbation methods.

B Data Appendix

This section describes the micro-data and the macro-data used in this papers. Section B.1 describe the steps for computing the micro-moments and B.4 describes the steps for computing the macro-moments.

B.1 GMM Micro-Moments Computation From COMPUSTAT

Table 1 describes the mnemonic for capital, output, debt and employment in COMPUSTAT. We computed a employment, capita, output and debt in the following way:

- **Employment (n)**: $n_{ti} = emp_{ti}$ and we define employment growth as

$$\Delta n_{ti} = \frac{emp_{ti} - emp_{t-1i}}{0.5 * (emp_{ti} + emp_{t-1i})} \tag{B.4}$$

- **Capital (k)**: $k_{ti} = ppent_{ti}$ and we define capital growth as

$$\Delta k_{ti} = \frac{(capx - sppe)_{ti}}{0.5 * (ppent_{ti} + ppent_{t-1i})} \tag{B.5}$$

- **Output (y)**: $y_{ti} = sale_{ti}$ and we define capital growth as

$$\Delta y_{ti} = \frac{sale_{ti} - sale_{t-1i}}{0.5 * (sale_{ti} + sale_{t-1i})} \tag{B.6}$$

- **Debt (b)**: $b_{ti} = \max\{dlc_{ti} + dltt_{ti} - che_{ti}, 0\}$ and we define capital growth as

$$\Delta b_{ti} = \frac{(dltis - dltr + ustdnc - chech)_{ti}}{0.5 * (b_{ti} + b_{t-1i})} \tag{B.7}$$

We apply the following filters to compute each moment:

- We drop observations with negative total assets, negative capital or negative sales.
- We drop firms with less than 5 years.
- For each $x \in \{b, n, k, y\}$, we construct a renormalize growth rate given by

$$\Delta \tilde{x}_{ti} = \Delta x_{ti} - \sum_i \frac{\Delta x_{ti}}{I(t)} - \sum_t \frac{\Delta x_{ti}}{T(i)} \tag{B.8}$$

- We drop $\pm 1\%$ outliers in employment, capital, debt and sales growth.

The mean, median, standard deviation, skewness, kurtosis and correlations with debt are in growth rate and persistence is in log-levels.

Table 1: Raw Data Short Description

Label	Short description	source	frequency	unit of measure	link
	Firms' identifier				
permco	firms' identifier	COMPUSTAT/CRSP	-	-	(1)-(2)
permno	security firms' identifier	COMPUSTAT/CRSP	-	-	(1)-(2)
gvkey	COMPUSTAT firms' identifier	COMPUSTAT	-	-	(2)
naics	North America Ind. Classification code	COMPUSTAT	-	-	(2)
	Firms' Balance Sheet Information				
atq/at	Asset Total	COMPUSTAT	Q/A	Millions of dollars	(1)-(2)
dlcq/dlc	Debt in Current Liabilities	COMPUSTAT	Q/A	Millions of dollars	(1)-(2)
dlttq/dltt	Long-Term Debt (LTD) - Total	COMPUSTAT	Q/A	Millions of dollars	(1)-(2)
dltis(y)/dltr(y)	LTD - issuance / reduction	COMPUSTAT	Q/A	Millions of dollars	(1)
dlcq/dlc	Debt in Current Liabilities (STD) - Total	COMPUSTAT	Q/A	Millions of dollars	(1)-(2)
ustdncy/ustdnc	STD net change	COMPUSTAT	Q/A	Millions of dollars	(1)
lctq/lct	Current Liabilities - Total	COMPUSTAT	Q/A	Millions of dollars	(1)-(2)
ltq/lt	Liabilities - Total	COMPUSTAT	Q/A	Millions of dollars	(2)
cheq/che	Cash and Short-Term Investments (STA)	COMPUSTAT	Q/A	Millions of dollars	(1)-(2)
chechq/chech	STA net change	COMPUSTAT	Q/A	Millions of dollars	(1)
ppentq/ppent	Prop., Plant, and Equip. (Net)	COMPUSTAT	Q/A	Millions of dollars	(1)-(2)
capx(y)/sppe(y)	Capital expenditure/Sale of property	COMPUSTAT	Q/A	Millions of dollars	(1)
	Firms' Equity Information				
prc	Closing price or ave. for a day	CRSP	D	raw price	(3)
shrout	Share outstanding	CRSP	D	Thousands of units	(3)
ret /retx	Returns with and without dividends	CRSP	D	-	(3)
	Firms' rest of variables and Aggregate Variables				
emp	Number of people employed	COMPUSTAT	A	Thousand	(1)
rf	Treasury constant maturity 1-year	CRSP	Daily	-	(4)
niq/ni	Net income (loss)	COMPUSTAT	Q/A	Millions of dollars	(1)-(2)
oib(a)dp(q)	Operating income (after)before depreciation	COMPUSTAT	Q/A	Millions of dollars	(1)-(2)
spcsrc	S&P Common Stock Ranking	COMPUSTAT	Q/A	Leters	(1)-(2)
saleq/sale	sales/turnover (NET)	COMPUSTAT	Q/A	Millions of dollars	(1)-(2)

In column frequency, D for daily frequency; Q stand for quarterly frequency; and A stand for annual frequency.

Links: (1) https://wrds-web.wharton.upenn.edu/wrds/ds/crsp/ccm_a/funda/index.cfm?navId=120;
(2) https://wrds-web.wharton.upenn.edu/wrds/ds/crsp/ccm_a/fundq/index.cfm?navId=120;
(3) https://wrds-web.wharton.upenn.edu/wrds/ds/crsp/stock_a/dsf.cfm?navId=128;
(3) https://wrds-web.wharton.upenn.edu/wrds/ds/frb/rates/index.cfm?navId=207;

Table 2: Micro-Moments From COMPUSTAT-Annual Frequency

moment	Debt	Employment	Sales	Capital
Micro-Moments				
mean	-0.001	0.000	0.001	-0.001
median	-0.026	-0.004	-0.004	-0.018
Standard Dev.	0.450	0.231	0.294	0.197
Skewness	0.223	0.082	0.101	0.996
Kurtosis	4.954	6.331	7.400	6.159
mean [w]	0.015	0.003	0.005	-0.001
median [w]	0.002	-0.002	-0.002	-0.004
Standard Dev. [w]	0.365	0.151	0.181	0.099
Skewness [w]	0.259	0.561	0.475	0.765
Kurtosis [w]	6.975	11.287	11.102	11.220
Corr. with Debt	1.000	0.167	0.105	0.200
Corr. with Debt [w]	1.000	0.188	0.069	0.173
Persistence	-0.108	-0.003	0.046	0.549
Persistence [w]	-0.062	0.042	0.120	0.432
Macro-Moments				
Standard Dev.	0.056	0.016	0.039	0.075
Correlation with debt	1.000	0.236	0.438	0.684
Persistence	0.185	0.190	0.077	0.204

This table describes firm level moments and aggregate moments computed from COMPUSTAT at annual frequency. For aggregate business cycle moments we use HP filter with smooth parameter 6.25 of the sum across firms. The weighted moments are with share of total assets (at in COMPUSTAT) and [w] denotes the weighted moment.

B.2 Merging CRSP with COMPUSTAT

- For annual data and daily data we generate a variable of date referring to the quarterly time. With this variable and permco we did the merge. For some firms, one permco is associated to several gvkeys of repeted data. We dropped duplicated observations whenever one permco is associated with several gvkeys.
- We clean the data base from financial firms ($sic \in [6000, 6999]$), regulated firms ($sic \in [4900, 4999]$) and public service firms ($sic \in [9000, \infty]$) for annual and quarterly data.
- For treasury rate drop date before 1965 and and write the rate of return on a daily bases.
- Merge annual COMPUSTAT with quarterly COMPUSTAT using date (quarterly) and permco.
- Use linear interpolation for going from annual to quarterly data. Use backward interpolation but never forward interpolation. Use these for two variables: employment and labor cost (emp and xstf).
- Scale balance sheet data by 1000000 and emp by 1000.
- Generate a balance panel for daily market value using linear interpolation for missing values.
- For the CRSP daily generates market value (watch out with negative values of prices), complete the missing values of market value using linear interpolation and collapse using sum by permco market value.
- Merge daily CRSP market value with compustat using permco and quarterly date. Then merge with daily treasury.

B.3 Estimation of the Merton Model

- Generate liaval using dsttq+0.5 dlttq
- Define L as liabilities; E as market value; r_t daily risk free rate.
- Use an iteration method to compute the value of the firm and distance to default.

  1. Guess and initial value for V:= assets values.
  2. Compute the mean and variance of the rate of return using V over a rolling window T.
  3. Compute the following objects:

$$\delta_{1t} = \frac{1}{\sigma_{Vt}\sqrt{dT}}(log(V_t/L_t) + (r_t * dT + \sigma_V^2 * dT * 0.5)) \tag{B.9}$$

$$\delta_{2t} = \delta_{1t} - \sigma_V \sqrt{dT} \tag{B.10}$$

$$N_{1t} = \Phi(\delta_{1t}) \tag{B.11}$$

$$N_{2t} = \Phi(\delta_{2t}) \tag{B.12}$$

$$V_t' = \frac{E_t + L_t * exp(r_t * dT * N_{2t})}{\sigma_{Vt}\sqrt{dT}} \tag{B.13}$$

4. Check if $V' - V$ is small. If it is small go to step 2 and there is only one iteration of step 4, stop. If it is small and there is more than one iteration of step 4 go to 2. If it is big, go to step 4.

- Compute distance to default using the following:

$$DD_t = \frac{1}{\sigma_{Vt}\sqrt{dT}}(log\left(\frac{V_t}{L_t}\right) + \mu_V dT - 0.5 * \sigma_{Vt} dT)) \tag{B.14}$$

$$Pr_t = \Phi(-DD_t) \tag{B.15}$$

B.4 Sources and Data Description of Aggregate Macroeconomic Variables

Table 3 describes the data sources for US macroeconomic time series together with the mnemonic for each variable. We follow Shimer (2005) to construct separation and finding probabilities; and Gilchrist and Zakrajsek (2012) credit spread time series. Monthly variables are transformed to quarterly frequency using averages. We construct additional variables using the following formulas:

- **Finding Rate**: $f_t = 1 - \frac{U_{t+1} - Ur_t}{U_t}$
- **Separation Rate**: $s_t = Ur_{t+1}/(E_t(1 - f_t/2))$

Business cycle components are obtained from a HP filter with smooth parameter 1600. TFP is constructed with the Solow residuals using business cycle fluctuation of TFP, output, labor and investment with the following formulas

$$T\hat{F}P_t = G\hat{D}P_t - 0.66 * \hat{L}_t - 0.33 * \hat{K_{t-1}} \quad ; \quad \hat{K}_t = (1 - 0.0025)\hat{K}_{t-1} + 0.0025\hat{I}_t \quad ; \quad \hat{K}_{-1} = 0 \tag{B.16}$$

Leverage is computed from COMPUSTAT using the following formula (see table 1 for the definition of each variable)

$$lev_t = \frac{\max\{DLCQ_{ti} + DLTTQ_{ti} - CHEQ_{ti}, 0\}}{PPENTQ_{ti}} \tag{B.17}$$

where $DLCQ$ and $DLTTQ$ denotes short and long term debt respectively, CEHQ denotes liquid assets and PPENTQ denotes capital. i denotes the ith-firm and time denotes time.

Aggregate debt is from Flow of Funds, Table Z1 "Financial Accounts of the United States". In particular, we use Table L.103 (Non-Financial Corporate Business, Quarterly) and constructed aggregate private debt as the sum of *Commercial Paper*, *Corporate Bonds*, *Loans from Depositary Institutions*, *Other Loans and Advances* and *Total Mortgages*.

Table 3: Macroeconomic Time Series — Description and Sources –

Label	**Short description**	**Source**	**Frequency-Adjustment**
GDP	Real Gross Domestic Product, Chained Dollars	NIPA[a]	Quarterly, SA
I	Gross Private Domestic Investment	NIPA	Quarterly, SA
DFF	Effective Federal Funds Rate, Percent	FRED[b]	Quarterly, SA
GDPDEF	Gross Domestic Product: Implicit Price Deflator	FRED	Quarterly, SA
E	Employment Level, Thousands of Persons	BLS	Monthly, SA (LNS13000000)
U	Unemployment Level, Thousands of Persons	BLS	Monthly, SA (LNS13000000)
Ur	Number Unemployed < 5 Weeks, Thousands of Persons	BLS	Monthly, SA (LNS13008396)
N	Population Level (1976-2016)	BLS	Monthly (LFU800000000)
N	Population Level (1948-1976)	BLS	Monthly (LNS10000000)
H	Average Weekly Hours Worked	BLS	Quarterly, SA(PRS85006023)
B	Debt for non-financial corporate business (1951-2016)	Flow of Funds[c]	Quarterly, NSA (Table Z1 - L.103)
gz-spread	GZ-credit spread I	GZ-AER (2012)[d]	Quarterly, NSA
gz-ebp	GZ-excess bond premium	GZ-AER (2012)	Quarterly, NSA
pi	Price of capital	SU-RED (2011) [e]	Quarterly, SA

NSA denotes non-seasonally adjusted and SA denotes seasonally adjusted

[a] http://www.bea.gov/iTable/iTable.cfm?ReqID=9#reqid=9&step=3&isuri=1&903=6
[b] https://research.stlouisfed.org/fred2
[c] http://www.federalreserve.gov/datadownload/
[d] https://www.aeaweb.org/articles?id=10.1257/aer.102.4.1692
[e] http://www.columbia.edu/~mu2166/common_trend/paper.pdf

Table 4: Business Cycle Moments - Annual Frequency

moment	Y	I	Emp	B
mean		0.142		
Std*100	0.014	0.065	0.011	0.024
$Std(x)/Std(Y)$	1.000	4.537	0.764	1.651
Autocorrelation	0.177	-0.007	0.297	0.501
Corr. with output	1.000	0.844	0.840	0.558

Y = Gross Domestic Output; I =Investment; Emp = number of employed; B = Debt for Non-Financial Corporate Firms. Mean investment is the mean of the ration investment-output. The variables in the data are annual from 1954 to 2015 with a Hodrick-Prescott filter with a smooth parameter of 6.25.

C Proofs

Proposition 1 (Equivalence Result) *The firm's problem in (8) and the surplus maximization problem in (12) are equivalent in the following sense*

(i) The surplus and the firm's value satisfy

$$\mathcal{S}(\Psi, s, F) = \mathcal{J}\left(\Psi, s, F, \{W_j\}_{j\in[0,n]}\right) + \int_0^n \left(W_j - \mathcal{U}(\Psi)\right) dj$$

(ii) If a policy $\left\{i, \hat{b}', \hat{n}, m, \varphi, \chi'\right\}$ maximizes the surplus, then there exist wages $\{w_j\}$ and continuation values $\{W'_j\}$ such that a contract $\omega_j = \left\{w_j, \varphi, \chi', W'_j\right\}$ is consistent with the promise keeping constraint (7), and $\left\{i, \hat{b}', \hat{n}, m, \{\omega_j\}\right\}$ solves the firm's problem and achieves value $\mathcal{J}\left(X, s, F, \{W_j\}_{j\in[0,x]}\right)$.

(iii) Conversely, if a policy $\left\{i, \hat{b}', \hat{n}, m, \{\omega_j\}\right\}$ maximizes the firm's problem; then $\left\{i, \hat{b}', \hat{n}, m, \varphi, \chi'\right\}$ solves the surplus' problem and achieve $\mathcal{S}(\Psi, s, F)$.

We will show this results with three lemmas than shows parts i-ii of the proposition. First we will show the part (i) of the theorem.

Lemma 1 *Let $\mathcal{S}(\Lambda, s, F)$ be the solution of 12 and $\mathcal{J}\left(\Lambda, s, F, \{W_j\}_{j\in[0,N]}\right)$ the solution of 8 , then*

$$\mathcal{J}\left(\Lambda, s, F, \{W_j\}_{j\in[0,N]}\right) = -\int_0^N W_j dj + \mathcal{U}(X)N + S(\Lambda, s, F) \tag{C.18}$$

Proof. Coming soon ■

Proposition 2 (Firm's policies) *A firm finds it optimal to default if and only if its cost shock is above a threshold $\underline{\boldsymbol{F}}(X, s)$ given as*

$$\underline{\boldsymbol{F}}(\Psi, s) = \eta z k^{\alpha\nu} n^{(1-\alpha)\nu} - \hat{u}(\Psi)n - F_k k - [(1-\tau)c + \lambda]\, b + v(\Psi, s) \tag{15}$$

where $v(\Psi, s) = \mathcal{S}(\Psi, s, F) - \mathcal{P}$. Then, a firm with state (Ψ, s) defaults with probability $1 - G\left(\underline{\boldsymbol{F}}(\Psi, s)\right)$.

If a firm does not default, its policies for investment, vacancy posting and debt issuance are independent of the cost shock. Denote $\boldsymbol{k}'(\Psi, s)$, $\boldsymbol{b}'(\Psi, s)$ and $\boldsymbol{n}(\Psi, s)$ the resulting states for capital, debt and workers next period, respectively.

Proof. Coming soon ■

D A Walrasian Market Model

In the models with walrasian market there are three agents: workers, investors and firms. Preferences, technology and debt contracts are the same as the model of search frictions, we only change the frictional labor market with walrasian labor where the firms take as given a fixed wage. Next, we will describe each agent problem.

- **Firms**: let $\mathcal{F}(\Psi, s, F)$ be the value of the firms with state $s = (b, k, n, z)$ when the aggregate state is Ψ and the fixed cost is F. Then

$$
\begin{aligned}
\mathcal{F}(\Psi, s, F) &= \max_{d,i,n,\hat{b}'} \left\{ d + \beta \mathbb{E}_{X'} \left[\left\{ \mathcal{F}(X', s', F') \right\}^+ | X, z \right] \right\} && \text{(D.19)} \\
\text{subject to} \quad \mathcal{P} &= \eta z k^{\alpha\nu} n^{(1-\alpha)\nu} - wn - F_K k - F - \left[(1-\tau)c + \lambda \right] b \\
\pi &= -\xi i + Q(b', k', n, \Psi, z)\hat{b}' - \mathcal{C}^K(i, k) - \mathcal{C}^N(n, n_-) - \mathcal{C}^B(b', b) \\
d &\leq \mathcal{P} + \pi \\
k' &= I + (1 - \delta_K)k, \qquad b' = (1 - \lambda)b + \hat{b}'
\end{aligned}
$$

  where $\mathcal{C}^N(N', N)$ is the adjustment cost on labor and $w(X)N$ is the total labor cost.

- **Investors**: let $\mathcal{I}(\Psi, \omega)$ be the value of an investor with wealth ω and aggregate state Ψ. Then

$$
\mathcal{I}(\Psi, \omega) = \max_{\{C, b'(h,z)\}} \left\{ C + \beta \mathbb{E}_{\Psi', F'} \left[\mathcal{I}\left(X', \omega'\right) \middle| \, \Psi \right] \right\} \qquad \text{(D.20)}
$$

subject to

$$
\begin{aligned}
\omega &\geq C + \frac{1}{\psi} \int_{\tilde{s}} Q(h', \Psi, z) b'(\tilde{s}) d\tilde{s} - \int d(s) d\mu(s) \\
\omega' &= \int_{\tilde{s}, s'} (1 - \chi(\Psi', s', F'))(c + \lambda + (1 - \lambda) Q(h'(\cdot), X', z')) b'(\tilde{s}) d\mu(s'|\tilde{s}) d\tilde{s} \\
&+ \int_{\tilde{s}, s'} \chi(\Psi', s', F') \left(Rk' \frac{b'(\tilde{s})}{b'} \right) d\mu(s'|\tilde{s}) d\tilde{s}
\end{aligned}
$$

E The Financial Value of a Worker

E.1 Data Description

The definition for the variables in this sections are:

- *Total Assets*: This variable defined as the log of total assets (atq) in COMPUSTAT.
- *Total Debt*: This variable defined as the log of debt in current liabilities, plus long-term debt minus liquid assets ($dlcq + dlttq - cheq$) in COMPUSTAT. If this variable is negative, we replace it with a missing value.
- *Total normalized profits*: This variable defined as the the ratio between operating income before depression and total asset ($oibdpq/atq$) in COMPUSTAT.
- *Investment rate*: This variable is defined as capital expenditure minus sales of property, divide the mean capital in the last two periods ($\frac{capxy_t - sppey}{0.5*(ppentq_t + ppentq_{t-1})}$) in COMPUSTAT.
- *Capital*: This variable defined as the log of total physical assets ($ppentq$) in COMPUSTAT.
- *Normalized Sales*: This variable defined as the log of the ratio between total sales and total cpaital ($saleq/ppentq$) in COMPUSTAT.
- *Liquid assets*: This variable defined as the log of liquid assets ($cheq$) in COMPUSTAT.

E.2 Robustness

This section in the online appendix discusses potential model misspecification that could implied a bias in our estimation above, and how to control for these. We show that our results are robust to controlling for these misspecification: a firm's number of workers significantly increase as its market value and decreases its probability to default.

In particular, we estimate a regression of the following form

$$\ln y_{it} \quad = \quad \alpha_i^y + \gamma_t^y + \beta^y X_{it} + \vartheta_N^y \ln N_{it} + \zeta_{it}^y \tag{E.21}$$

where y_{it} can either be the market value of a firm, or its probability of default. We interpret the coefficient ϑ_N as the *marginal* effect that employment has on y_{it} after controlling for firms' states X_{it}. [27] The estimate for ϑ_N^y will naturally depend on what variables we decide to include as controls in X_{it}. Unlike in previous sections, we include additional controls in X_{it} to account for alternative model misspecification. Table 6 describe the robustness exercises with probability of default; table 5 the robustness exercises with market value; and table 7 the robustness checks with the logic of the probability of default.

A first source of bias could arise from a misspecification of the persistence component of productivity. For instance, if firm's productivity depends on several of its lags, this period profits and investment would not contain

[27] The log-log nature of equation (E.21) allows to interpret ϑ_N as an elasticity.

enough information about the firm's future profitability. In this case, market value and number of workers could exhibit a positive relation because both respond to an information about productivity not included in our controls. In order to control for this source of bias, we add lags of profits to our controls. We find that the value of a worker is robust to this additional controls.

A second source of bias could arise if firms, or market participants, have some information or signals about future firms' profitability. Good news about the future, for instance, could induce a firm to hire more and simultaneously experience an increase in its market value. This would imply our found correlation, but because of a different mechanism. In order to control for this, we include future profits in our controls.

A third source of bias could arise if the firms assets composition affects the firm's value. For instance, a very illiquid firm may experience a decrease in its market value, and persistently lower hiring rates. In order to control for this source of bias, we include several measures of the firms portfolio as controls, such as; total assets, physical assets and liquid assets. We find that the value of a worker is robust to this additional controls.

A final source of potential bias could from the existence of *customer capital.* For instance, higher sales today may enlarge the firm's costumers base and increase future profitability. In this case, a firm's market value and its number workers may exhibit a positive correlation even after controlling for the firm's assets. In order to control for this source of bias, we add total sales as a control variable, which are not perfectly correlated with number of workers. Again, we find that the value of a worker is robust to this additional controls.

The computations we have done so far estimate the average effect that a marginal worker has on the firm's market value or its probability to default. However, this effect could in principle vary with the firm's size. To test robustness across firms size distribution, we divides firms into quantiles depending either on the firm's assets size or employment size, and repeats the previous exercises. We find evidence of the *unemployment accelerator* across the spectrum of firms size distribution.

As a result of these robustness checks, we conclude that there exist a significant positive relation between the value of a firm and its number of workers. Similarly, there is a significant negative relation between the expected probability of default and the number of workers. These results are robust across several features in the data that our model abstracts from. This suggests that workers act as an asset to the firm, in the same manner that capital does. This is the new fact we provide, together with a model that is capable of replicating this relation.

Table 5: Dependent Variable: Log of Market Value

Variable	I	II	III	IV	V
Workers	0.40^{***}	0.39^{***}	0.38^{***}	0.10^{***}	0.20^{***}
Capital	0.35^{***}	0.35^{***}	0.36^{***}	0.22^{***}	0.63^{***}
Assets	-	-	-	0.60^{***}	-
Debt	-0.06^{***}	-0.05^{***}	-0.06^{***}	-0.12^{***}	-0.08^{***}
Profits	0.22^{***}	0.22^{***}	0.33^{***}	0.07^{***}	0.05^{***}
Profits L	-	0.39^{***}	-	-	-
Profits L^2	-	0.68^{***}	-	-	-
Profits L^3	-	1.03^{***}	-	-	-
Profits F	-	-	0.39^{***}	-	-
Profits F^2	-	-	0.17^{***}	-	-
Profits F^3	-	-	0.09^{***}	-	-
Invest. rate	0.01^{***}	0.00^{**}	0.04^{***}	0.01^{***}	0.06^{***}
Liquid assets	-	-	-	0.01^{***}	-
Sales	-	-	-	-	0.33^{***}
N obs	160761	145249	142585	157896	159615
R^2	0.726	0.724	0.730	0.811	0.759

Table 6: Dependent Variable: Log of Probability of Default

Variable	I	II	III	Reg. IV	Reg. V		
Workers	-0.63^{***}	-0.56^{***}	-0.55^{***}	-0.37^{***}	-0.32^{***}		
Capital	-0.35^{***}	-0.34^{***}	-0.40^{***}	-0.24^{***}	-0.78^{***}		
Assets	-	-	-	-	-	-0.58^{***}	-
Debt	0.71^{***}	0.65^{***}	0.69^{***}	0.77^{***}	0.73^{***}		
Profits	-1.52^{***}	-1.43^{***}	-1.79^{***}	-1.37^{***}	-1.37^{***}		
Profits L -	-3.38^{***}	-	-	-			
Profits L^2 -	-3.43^{***}	-	-	-			
Profits L^3 -	-4.68^{***}	-	-	-			
Profits F	-	-	-1.83^{***}	-	-		
Profits F^2	-	-	-1.66^{***}	-	-		
Profits F^3	-	-	-0.81^{***}	-	-		
Invest. rate	-0.06^{***}	-0.04^{***}	-0.15^{***}	-0.06^{***}	-0.25^{***}		
Liquid assets	-	-	-	0.04^{***}	-		
Sales	-	-	-	-	-0.51^{***}		
N obs	154072	139199	138740	151358	153031		
R^2	0.204	0.234	0.217	0.239	0.237		

Table 7: Dependent Variable: Logic of Probability of Default

Variable	I	II	III	IV	V
Workers	-0.71^{***}	-0.65^{***}	-0.65^{***}	-0.42^{***}	-0.34^{***}
Capital	-0.41^{***}	-0.38^{***}	-0.48^{***}	-0.28^{***}	-0.91^{***}
Assets	-	-	-	-0.63^{***}	-
Debt	0.70^{***}	0.66^{***}	0.69^{***}	0.77^{***}	0.73^{***}
Profits	-0.94^{***}	-0.97^{***}	-1.45^{***}	-0.82^{***}	-0.68^{***}
Profits L	-	-1.86^{***}	-	-	-
Profits L^2	-	-3.20^{***}	-	-	-
Profits L^3	-	-4.82^{***}	-	-	-
Profits F	-	-	-1.55^{***}	-	-
Profits F^2	-	-	-0.87^{***}	-	-
Profits F^3	-	-	-0.79^{***}	-	-
Invest. rate	-0.04^{***}	-0.01	-0.10^{***}	-0.04^{***}	-0.17^{***}
Liquid assets	-	-	-	0.05^{***}	-
Sales	-	-	-	-	-0.61^{***}
N obs	160761	145249	142585	157896	159615
R^2	0.355	0.388	0.251	0.376	0.371