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Sovereign Default: The Role of Expectations*

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ABSTRACT

We study a variation of the standard model of sovereign default, as in Aguiar and Gopinath (2006) or Arellano (2008), and show that this variation is consistent with multiple interest rate equilibria. Some of those equilibria correspond to the ones identified by Calvo (1988), where default is likely because rates are high, and rates are high because default is likely. The model is used to simulate equilibrium movements in sovereign bond spreads that resemble sovereign debt crises. It is also used to discuss lending policies similar to the ones announced by the European Central Bank in 2012.

Keywords: Sovereign default; Interest rate spreads; Multiple equilibria
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1 Introduction

The focus of this paper is the origins of sovereign debt crises. Are sovereign debt crises caused by bad fundamentals alone, or do expectations play an independent role? The main point of the paper is that both fundamentals and expectations can indeed play important roles. High interest rates can be triggered by self-confirming expectations, but those high rates are more likely when debt levels are relatively high. The model analyzed can help to explain the large and abrupt increases in spreads during sovereign debt crises, particularly in countries that have accumulated large debt levels, as seen in the recent European experience. It can also justify the policy response by the European Central Bank, to be credited for the equally large and abrupt reduction in sovereign spreads.

The literature on sovereign debt crises is ambiguous on the role of expectations. In a model with rollover risk, Cole and Kehoe (2000) have established that sunspots can play a role that is strengthened by bad fundamentals. Using a different mechanism, Calvo (1988) also shows that there are both low and high interest rate equilibria. The reason for the multiplicity in Calvo is that, although interest rates may be high because of high default probabilities, it is also the case that high interest rates induce high default probabilities. This gives rise to equilibria with high rates/likely default and low rates/unlikely default. In contrast with the results in those models, in the standard quantitative model of sovereign default, as in Aguiar and Gopinath (2006) or Arellano (2008), there is a single low interest rate equilibrium.

In this paper, we take the model of Aguiar and Gopinath (2006) and Arellano (2008), which builds on the model of Eaton and Gersovitz (1981), and make minor changes in the modeling choices concerning the timing of moves by debtors and creditors and the actions that they may take. In so doing, we are able to produce expectation-driven movements in interest rates. The reason for those movements is the one identified by Calvo (1988) and more recently analyzed in Lorenzoni and Werning (2013).[1] The change in the modeling choices is minor because direct evidence cannot be used to discriminate across them. On the other hand, ample indirect evidence is provided by large and abrupt movements in spreads, apparently unrelated to fundamentals, during sovereign debt crises.

Our theoretical exploration of self-fulfilling equilibria in interest rate spreads is

[1] We started working independently. Their first version precedes ours.

motivated by two particular episodes of sovereign debt crises. The first is the Argentine crisis of 1998-2002. Back in 1993, Argentina had regained access to international capital markets. The debt-to-GDP ratio was roughly between 35% and 45% during the period—very low by international standards. The average yearly growth rate of GDP was around 5%. But the average country risk spread on dollar-denominated bonds for the period 1993-1999, relative to the US bond, was 7%. Notice that a 7% spread on a 35% debt-to-GDP ratio amounts to almost 2.5% of GDP on extra interest payments per year.[2] Accumulated over the 1993-1999 period, this is 15% of GDP, almost half the debt-to-GDP ratio of Argentina in 1993. An obvious question arises: if Argentina had faced lower interest rates, would it have defaulted in 2002?

The second episode is the recent European sovereign debt crisis that started in 2010 and has receded substantially since the policy announcements by the European Central Bank (ECB) in September 2012. The spreads on Italian and Spanish public debt, very close to zero since the introduction of the euro and until April 2009, were higher than 5% by the summer of 2012, when the ECB announced the program of Outright Monetary Transactions (OMTs). The spreads were considerably higher in Portugal, and especially in Ireland and Greece. With the announcement of the OMTs, according to which the central bank stands ready to purchase euro area sovereign debt in secondary markets, the spreads in most of those countries slid down to less than 2%, even though the ECB did not actually intervene. The potential self-fulfilling nature of the events leading to the high spreads of the summer 2012 was explicitly used by the president of the ECB to justify the policy.[3]

The model in this paper is of a small open economy with a random endowment, very similar to the structure in Aguiar and Gopinath (2006) or Arellano (2008). A representative agent can borrow noncontingent bonds and cannot commit to repayment. Defaulting carries a penalty. The borrower faces atomistic foreign creditors, each of which has limited funds. The creditors are risk neutral, so that the average return from lending to this economy, taking into account the probability of default, has to be equal to the risk-free international rate of interest. The timing and action assumptions are the following. In the beginning of the period, given the level of debt gross of interest and the realization of the endowment, the borrower decides whether to default. If there

[2] This calculation unrealistically assumes one-period maturity bonds only. Its purpose is only to illustrate the point in a simple way.

[3] The decision has raised controversy. In 2014, the German Constitutional Court ruled the OMT to be incompatible with the constitution.

is default, the endowment is forever low. Otherwise, creditors move first and offer their limited funds at some interest rate. The borrower moves next and borrows from the low-rate creditors up to some total optimal debt level. In equilibrium, the creditors all charge the same rate, which is the one associated with the probability of default for the optimal level of debt chosen by the country. With these timing assumptions, there are multiple interest rate equilibria. High interest rates can generate high default rates, which in turn justify high interest rates. In equilibria such as these, there is a sense in which interest rates can be "too high."

With this timing, when deciding how much to borrow, the borrower takes the interest rate as given. This does not mean that the borrower behaves like a small agent. Even if it takes current prices as given, it still takes into account the effects of its current choices on future prices. The borrower is simply not benefiting from a first mover advantage.[4]

The timing assumptions in Aguiar and Gopinath (2006) and Arellano (2008) are such that the borrower moves first, before the creditors. They also assume that the borrower chooses the debt level at maturity, including interest payments. Creditors move next and respond with a schedule that specifies a single interest rate for each level of debt gross of interest. By moving first and choosing the debt at maturity, gross of interest, the borrower is able to select a point in the schedule, therefore pinning down the interest rate. It follows that there is a single equilibrium. The first mover advantage allows the borrower to coordinate the creditor's actions on the low interest rate equilibrium.

An alternative structure has the same sequence of moves, except that the borrower chooses current debt instead of debt at maturity. The interest rate schedule will then be a function of current debt, rather than debt at maturity. In this case, there will in general be multiple schedules.[5] Given current debt, if the interest rate is high, so is debt at maturity, and therefore the probability of default is also high. This is the spirit of the analysis in Calvo (1988).

[4] A similar timing assumption in Bassetto (2005) also generates multiple Laffer curve equilibria. In Bassetto, if the government were to move first and pick the tax, there would be a single low tax equilibrium. Instead, if households move first and supply labor, there is also a high tax equilibrium. Bassetto convincingly argues that the assumption that the government is a large agent is unrelated to the timing of the moves.

[5] In Eaton and Gersovitz (1981), even if that is the assumption on the actions of the country, they dismiss the multiplicity by assumption (discussed further in Section 2.3).

Current debt in Calvo (1988) is exogenous, but debt at maturity is not because it depends on the endogeneously determined interest rate. If the borrower were to choose debt at maturity, given current debt, the interest rate would be pinned down, and, again, there would be a single equilibrium. Lorenzoni and Werning (2013) analyze a dynamic version of Calvo's model with exogenous public deficits and argue against the possibility of the government choosing debt at maturity. For that, they build a game with an infinite number of subperiods and assume that the government cannot commit not to reissue debt in those subperiods. As a result, the government is unable to select a point on the interest rate schedule.

As mentioned above, the reason for expectation-driven high interest rate equilibria in these models is different from the one in Cole and Kehoe (2000). Still, in that setup it is the timing of moves that is crucial to generating multiplicity. In Cole and Kehoe, there is multiplicity when the choice of how much debt to issue takes place before the decision to default. In that case, it may be individually optimal for the creditors not to roll over the debt, which amounts to charging arbitrarily high interest rates. This may induce default, confirming the high interest rates. In our model, there is no rollover risk because the decision of default is at the beginning of the period. Still, a timing assumption similar to the one in Cole and Kehoe generates the multiplicity. As creditors move first, it can be individually optimal to ask for high rates. That will induce a high probability of default, confirming the high rates.

The theoretical analysis in this paper is done in a two-period version of the model in which the intuition is very clear. We discuss the relevance of the alternative timing and action assumptions. The model is first solved with our preferred timing in which the borrower behaves like a price taker. The solution can be derived very simply using a demand curve of debt by the borrower and a supply curve of funds by the creditors. In general, there are multiple intersections of the demand and supply curves. These intersections are all potential equilibria, but some are more compelling than others.

For standard distributions of the endowment, the high rate equilibria have properties that make them vulnerable to reasonable refinements. Those high rates can be in parts of the supply curve in which the rates decrease with an increase in the level of debt. If that is the case, then the total gross service of the debt also decreases with an increase in the level of debt. For those high rates, creditors also jointly benefit from lowering interest rates because of their effect on probabilities of default. These are all features of the high rate equilibria in Calvo (1988). But as we show, multiplicity does not disappear even if those equilibria are refined away. To show this, we consider bimodal distributions for the endowment, with good and bad times. With those distributions, there are low and high rate equilibria, equally robust, for the same level of debt. The set of equilibria has the feature that for low levels of debt, there is only one equilibrium. Interest rates are low and increase slowly with the level of debt. As debt becomes relatively high, then there are both low and high rate equilibria. For even higher levels of debt, there is a single high rate equilibrium, until eventually there is none. As we explain in detail in the paper, we consider these binomial distributions as reflecting the likelihood of relatively long periods of stagnation, as currently discussed in Europe, in a way that resembles the Markov-switching processes for output popularized by Hamilton (1989).[6]

In the region where the interest rates are unnecessarily high, policy can be effective in selecting a low rate equilibrium. A large lender can accomplish the missing coordination by lending up to a maximum amount at a penalty rate. In equilibrium, only private creditors would be lending. This may help us to understand the role of policies such as the OMTs introduced by the ECB, following the announcement by its president that it would do "whatever it takes" to avoid a sovereign debt crisis in the euro area.

The paper also includes a quantitative section with a dynamic model in which a sunspot variable is introduced, which triggers coordination on high or low interest rates. To stay closer to the quantitative literature, and also for simplicity in the computations, we consider the standard timing in the literature that has the borrower move first and face an interest rate schedule. In order to have multiplicity, the schedules are in terms of debt net of interest. The model is shown to be consistent with a sovereign debt crisis unraveling, in particular when debt is relatively large - as is the case in Europe now - and the probability of a relatively long period of stagnation is high.

As mentioned, this paper is closely related in its motivation to Lorenzoni and Werning (2013), but there are some very important differences. First, Lorenzoni and Werning (2013) consider long maturity debt and focus the analysis on equilibria with debt dilution, whereas we do not. In our setup, the multiplicity is closer to the one analyzed by Calvo (1988): it arises only with short-term debt. We emphasize the role of large debt levels and the plausibility of long periods of stagnation as possible drivers of the multiplicity. Second, they study a model in which fiscal policy is exogenous. We

[6] See also the evidence in Jones and Olken (2008) for an international perspective.

instead characterize equilibria with optimal debt choices. In our model, the possibility of a long stagnation is key, so we view our results as illustrative of possibilities over low frequencies, and it seems natural to consider aggregate fiscal policy as endogenous in that case. Our choice has the advantage that we can discuss the role of the endogenous decision to borrow on the likelihood and characteristics of the debt crisis. This disciplinary effect of crisis is discussed in detail in our quantitative section. It also highlights a key role for quantity restrictions in the design of policies aimed at eliminating the "bad" equilibria, as the OMT discussed above, suggesting that to do "whatever it takes," understood as no limit on bond purchases, does not necessarily follow from the model. On the theoretical front, we only analyze a simple two-period model and highlight the importance of both timing and action assumptions for multiple interest rate equilibria to arise, clarifying apparent inconsistencies in the literature. By exposing the importance of these assumptions, we argue for the empirical relevance of that multiplicity. With a similar objective, but with a different approach, Lorenzoni and Werning (2013) analyze games that also provide support for multiplicity.

2 A two-period model

It is useful to first analyze the case of a simple two-period model, where analytical results can be derived and some of the features of the model can be seen clearly. In particular, it is easier to understand in the two-period model what drives the multiplicity of spreads and default probabilities that resembles the result in Calvo (1988).

We analyze a two-period endowment economy populated by a representative agent that draws utility from consumption in each period, and by a continuum of risk-neutral foreign creditors. Each creditor has limited capacity, but there are enough of them so that there is no constraint on the aggregate credit capacity. The period utility function of the representative agent, U, is assumed to be strictly increasing and strictly concave and to satisfy standard Inada conditions. The endowment is assumed to be equal to 1 in the first period. That is the lower bound of the support of the distribution of the endowment in the second period. Indeed, uncertainty regarding future outcomes is described by a stochastic endowment $y \in [1, Y]$, with density $f(y)$ and corresponding cdf $F(y)$. The outstanding initial level of debt is assumed to be zero, and in period one, the representative agent can borrow b in a noncontingent bond in international financial markets. The risk-neutral gross international interest rate is R^*. In period two, after observing the realization of the shock, the borrower decides to either pay the debt gross of interest, Rb, or default. If there is default, consumption is equal to the lower bound of the endowment process, 1. Note that there may be contingencies under which the borrower chooses to default, and the interest rate charged by foreign creditors, R, may differ from the risk-free rate R^*.

The timing of moves is as follows. In the first period, each creditor $i \in [0, 1]$ offers the limited funds at the gross interest rate R_i. The borrower moves next and picks the level of debt $b = \int_0^1 b_i di$, where b_i is how much is borrowed from each creditor. The borrower's best response is to borrow from the low interest rate lenders first. Since each creditor has limited funds, neither of them can individually affect the probability of default. Therefore, in order to make zero profits in equilibrium, the interest rates they charge will have to be the same, $R_i = R$. Without loss of generality, we focus on symmetric outcomes where if $R_i = R_j$, then $b_i = b_j$. Then, $b_i = b$ for all $i \in [0, 1]$, so $\int_0^1 b_i R_i di = a = Rb$.

In the second period, the borrower decides whether to default fully or pay the debt in full. The borrower decides to default if and only if $U\left(y - \int_0^1 b_i R_i di\right) \leq U(1)$, or

$$y \leq 1 + \int_0^1 b_i R_i di.$$

In order for creditors to make zero profits in equilibrium, the interest rates they charge will have to be the same, $R_i = R$. Assuming the country borrows the same amount from each creditor, default happens whenever

$$y \leq 1 + bR,$$

which defines a default threshold for output. The probability of default is then $F[1 + bR]$.

Since creditors are risk neutral, the expected return of lending to the borrower in this economy must be the same as R^*, so

$$R^* = R[1 - F(1 + bR)]. \tag{1}$$

This defines a locus of points (b, R) such that each point solves the problem of the creditors, which can be interpreted as a supply curve of funds. The mapping from debt levels to interest rates is a correspondence because, in general, for each b there are multiple Rs that satisfy equation (1). Multiple functions can be built with the points of the correspondence. We call those functions interest rate schedules.

The optimal choice of debt by the borrower is the one that maximizes utility:

$$U(1+b) + \beta \left[F(1+bR)U(1) + \int_{1+bR}^{Y} U(y-bR)f(y)dy \right], \tag{2}$$

subject also to an upper bound restriction on the maximum level of debt. Absent this condition, the optimal choice would be to borrow an arbitrarily large amount and default with probability one. The supply of debt would be zero in equilibrium.

The marginal condition, for an interior solution, is

$$U'(1+b) = R\beta \int_{1+bR}^{Y} U'(y-bR)f(y)\,dy. \tag{3}$$

The optimal choice of debt for a given interest rate defines a locus of points (b, R) that can be interpreted as a demand curve for funds. The possible equilibria will be the points where the demand curve intersects the supply curve above described by (1).

An equilibrium in this economy can then be defined as follows:

Definition 1 *An equilibrium is an interest rate $\widetilde{R}$ and a debt level $\widetilde{b}$ such that (i) given $\widetilde{R}$, $\widetilde{b}$ maximizes (2); and (ii) the arbitrage condition (1) is satisfied.*

2.1 Multiple equilibria

As mentioned above, there are in general multiple equilibria in this model—low rate equilibria and high rate equilibria—that resemble the multiple equilibria in Calvo (1988).

We now analyze the supply curve defined implicitly by (1). For that purpose, it is useful to define the function for the expected return on the debt:

$$h(R;b) = R\left[1 - F(1+bR)\right],$$

which must be equal to the riskless rate, R^*. For $R = 0$, $h(0;b) = 0$. If the distribution of the endowment has a bounded support, for R high enough, if $1 + bR \geq Y$, then $h(R;b) = 0$. For standard distributions, the function $h(R;b)$ is concave, so that there are at most two solutions for $R^* = h(R;b)$.

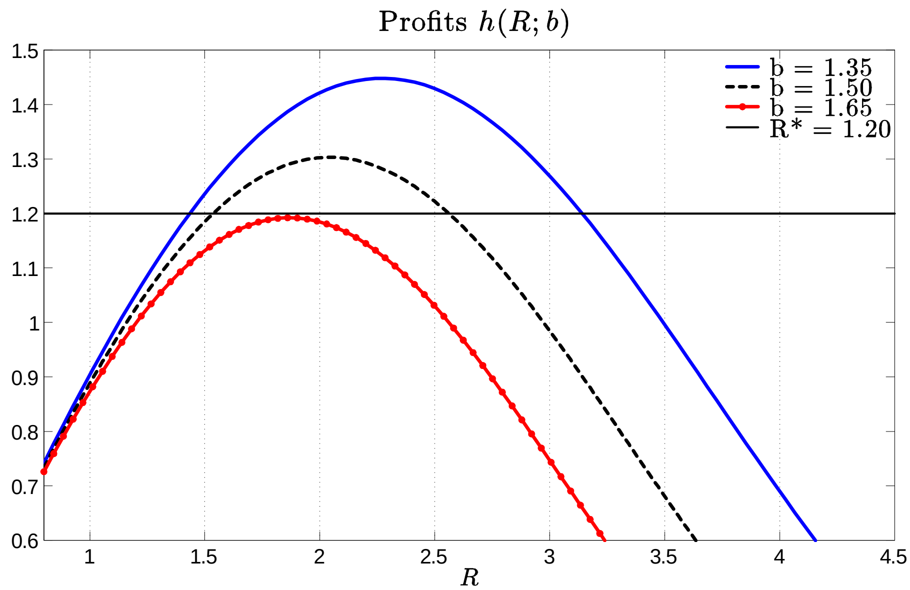

Figure 1: Expected return $h(R;b)$

In the case of the uniform distribution, it is straightforward to obtain the solutions of $R^* = h(R;b)$, so that the supply curve can be described analytically. Let the distribution of the endowment process be the uniform, $f(y) = \frac{1}{Y-1}$, so that $F(y) = \frac{y-1}{Y-1}$. Then, from (1), the equilibrium interest rates must satisfy

$$R = \frac{1 \pm \left(1 - 4\frac{R^* b}{Y-1}\right)^{\frac{1}{2}}}{2\frac{b}{Y-1}},$$

provided $1 - 4\frac{R^* b}{Y-1} \geq 0$. The maximum level of debt consistent with an equilibrium with borrowing is given by $b^{\max} = \frac{Y-1}{4R^*}$. Below this value of debt, for each b, there are two possible levels of the interest rate.

In Figure 1, the curve $h(R;b)$ is depicted against R, where F is the cumulative normal.[7] An increase in b shifts the curve h downward so that the solutions for b are closer to each other. The second derivative of $h(R;b)$ is negative when $2f(1+bR) \geq -f'(1+bR)\, bR$. The function $h(R;b)$ does not have to be concave everywhere. This depends on the cumulative distribution $F(1+bR)$. Later we discuss conditions for the nonconcavity of the function $h(R;b)$.

[7] The black vertical dotted lines are grid lines. We kept them in the plots throughout the paper to make the exposition clearer.

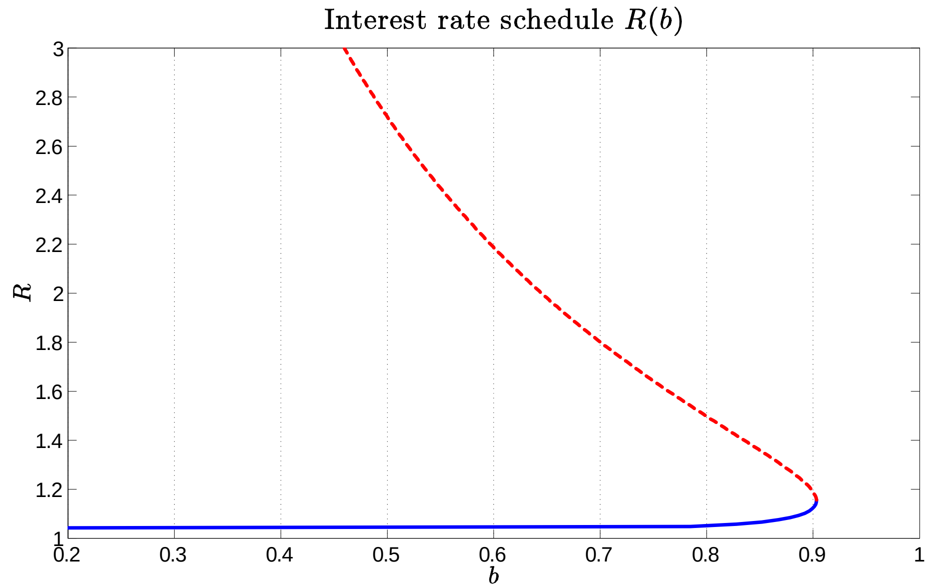

Figure 2: Interest rate schedules

Figure 2 plots the solutions for R of equation (1) for each level of debt and also for the normal distribution.

The supply curve of Figure 2 has two monotonic schedules. For lower values of the interest rate, there is a flat schedule that is increasing in b (solid line). There is also a steeper decreasing schedule for higher values of the interest rate (dashed line).

The equilibrium must also be on a demand curve for the borrower, obtained from the solution of the problem defined in (2). Figure 3 depicts the two curves: the supply curve (red) and the demand curve (blue).

The points on the decreasing schedule have particularly striking properties. For those points on the supply curve, not only does the interest rate go down with the level of debt, b, but also the gross service of the debt, Rb, decreases with the level of debt, b. To see this, notice that from (1), R increases with the level of Rb. The points on the decreasing schedule are weak candidates for equilibria in the following sense. Consider a perturbation of a point $(\widehat{R}, \widehat{b})$ in that schedule that consists of the same value for the interest rate but a slightly lower value for debt $(\widehat{R}, \widehat{b} - \varepsilon)$. This point would lie below the schedule. At the point $(\widehat{R}, \widehat{b} - \varepsilon)$, the interest rate is the same as in $(\widehat{R}, \widehat{b})$, but the debt is lower, so the probability of default is also lower. Thus, profits for the creditors are higher than at $(\widehat{R}, \widehat{b})$, where profits are zero. This means that a small reduction in the interest rate is beneficial for both the borrower and the lender, which

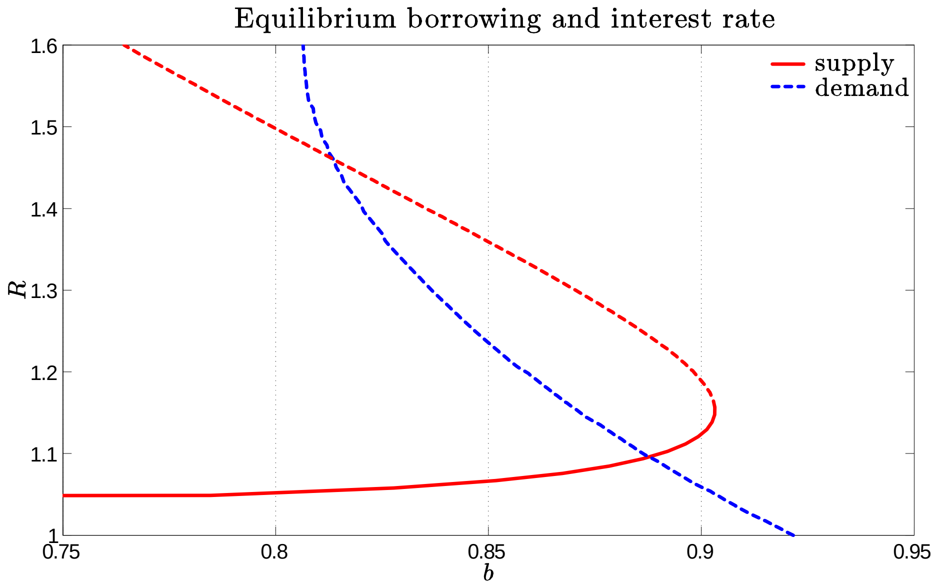

Figure 3: Supply and demand curves

suggests that these equilibria may not survive reasonable refinements. In Appendix 1 we perturb the extensive form game in the first period and offer a refinement that indeed rules out equilibria on the decreasing schedules.[8] One could then hope that, subject to this refinement, the equilibrium would therefore be unique. As we now show, such hopes are not realized.

2.1.1 A distribution with good and bad times

Equation (1) may have more than two solutions for R, for a given b, depending on the distribution of the endowment process. One case in which there can be multiple increasing schedules is when the distribution combines two normal distributions—a distribution for good times and a distribution for bad times.

Consider two independent random variables, y^1 and y^2, both normal with different means, μ^1 and μ^2, respectively, and the same standard deviation, σ. Now, let the endowment in the second period, y, be equal to y^1 with probability p and equal to y^2 with probability $1 - p$.

[8] There are two important assumptions, as we explain in detail in Appendix 1. First, there must exist a minimal degree of coordination, which, for some equilibria in the decreasing schedule, may be large. Second, the first-period auction must be anonymous, in the sense that ex ante differences that arise because of the perturbation cannot be observed by the borrower.

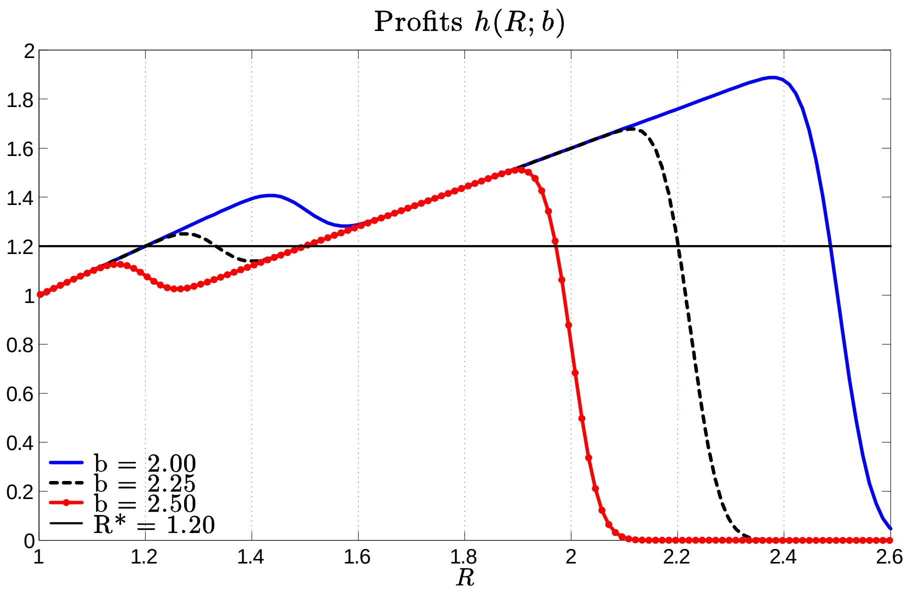

Figure 4: Expected return for the bimodal distribution $h(R;b)$

If the two means, μ^1 and μ^2, are sufficiently apart, then (1) has four solutions for some values of the debt, as Figure 4 shows. The correspondence between levels of debt and R, as solutions to the arbitrage equation above, is plotted in Figure 4, in which $p = 0.8$, $\mu^1 = 4$, $\mu^2 = 6$, and $\sigma = 0.1$.[9] Clearly, there are debt levels for which there are only two solutions, so there is only one increasing schedule. But for intermediate levels of debt, the equation has four solutions and therefore multiple increasing schedules. This means that, even if one is restricted not to consider equilibria on decreasing schedules, the model may still exhibit multiplicity. Notice that the multiplicity on the increasing schedules arises for relatively high levels of debt.

The supply curve for this case of the bimodal distribution is indicated by the solid red line in Figure 5. The demand is shown by the dotted blue line in the same figure. Notice that multiplicity only arises if the demand curve is high enough, so the resulting equilibrium level of debt is high. The demand is discontinuous in this case, since the maximum problem in (2) has two interior local maxima, because of the bimodal distribution. As the interest rate changes, the relative value of utility between the two local maxima changes.[10]

[9] The relatively high probability and the average severity of a disaster can be thought of as a relatively frequent, long period of stagnation. We explore this alternative in the numerical section.

[10] The details are in an appendix available upon request. Note that theoretically, it would be possible

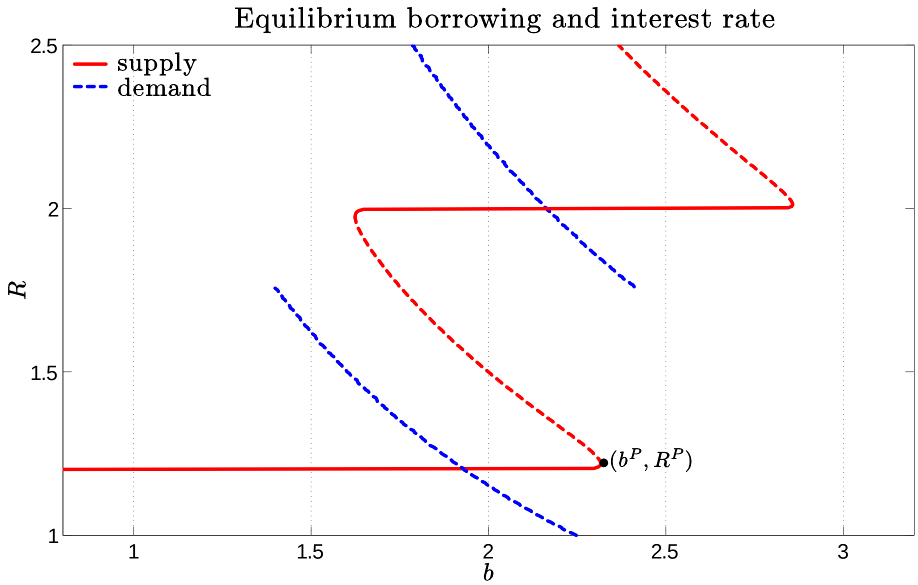

Figure 5: Supply and demand for the bimodal distribution

If the debt level is relatively large, multiple equilibria are more likely to arise. This is the case with the bimodal distribution analyzed earlier, but it is particularly so when the value of the debt is close to the maximum and any single mode distribution is perturbed by adding a nonmonotonic transformation. The details are in Appendix 2.

2.2 Policy

To illustrate the effects of policy, the case of the bimodal distribution depicted in Figure 5 is considered. The extensions to other cases are straightforward.

Consider there is a new agent, a foreign creditor that can act as a large lender, with deep pockets.[11] This large lender can offer to lend to the country, at a policy rate R^P, any amount lower than or equal to a maximum level b^P. It follows that there cannot be an equilibrium with an interest rate larger than R^P.

Now, let us imagine that b^P and R^P are the debt level and interest rate correspond-

to have multiplicity with low levels of debt arising simply from the discontinuity of the demand. This never happened in our simulations, however.

[11] If the borrower was a small agent rather than a sovereign, any creditor could possibly play this role.

ing to the maximum point of the low (solid line) increasing schedule in Figure 5. In this case, the only equilibrium is the point corresponding to the intersection of demand and supply on the low interest rate increasing schedule. In addition, the amount borrowed from the large lender is zero. The equilibrium interest rate is lower than the one offered by the large lender because at that interest rate R^P and for debt levels strictly below b^P, there would be profits.

Notice that the large lender cannot offer to lend any quantity at the penalty rate. Whatever the rate is, the level of lending offered has to be limited by the points on the supply curve. Otherwise, the borrower may borrow a very high amount and then default.

2.3 Current debt versus debt at maturity

The borrower in the model analyzed earlier chooses current debt. Would it make a difference if the borrower were to choose debt at maturity, gross of interest? We now consider an alternative game in which the timing of the moves is as before, but now the borrower chooses the value of debt at maturity, which we denote by a, rather than the amount borrowed, b. Are there still multiple equilibria in this setup? The answer is yes. With this timing of moves, there are multiple interest rate equilibria whether the government chooses the amount borrowed, b, or the amount paid back, a. This is a relevant question, because in the models of Calvo (1988) and Arellano (2008), the assumption of whether the borrower chooses b or a is key to having uniqueness or multiplicity of equilibria, as will be discussed later.[12]

Here again, the creditors move first and offer the limited funds at gross interest rate R_i, $i \in [0,1]$. The borrower moves next and picks the level of debt at maturity $a = \int_0^1 a_i di$. As before, the rate charged by each creditor will have to be the same in equilibrium. In the second period, the borrower defaults if and only if $y \leq 1 + a$. Arbitrage in international capital markets implies that

$$R^* = R\left[1 - F\left(1 + a\right)\right]. \tag{4}$$

The locus of points (a, R) defined by (4), which we interpret as a supply curve of funds, is monotonically increasing (which is not the case for the supply curve in b and

[12]The key for the different results is the timing assumption, as clarified in Section 3.

R defined in (1)).

The utility of the borrower is

$$U(1+\frac{a}{R})+\beta\left[F(1+a)U(1)+\int_{1+a}^{Y}U(y-a)f(y)dy\right], \tag{5}$$

where $\frac{1}{R}$ is the price of one unit of a as of the first period. The marginal condition is

$$U'(1+\frac{a}{R})=R\beta\int_{1+a}^{Y}U'(y-a)f\left(y\right)dy. \tag{6}$$

The locus of points (a, R) defined by the solution to this maximization problem can be interpreted as a demand curve for funds. Again, this demand curve with the supply curve has multiple intersection points. Provided the choice of a is interior, those points are the solutions to the system of two equations, (4) and (6), but those are the same two equations (1) and (3) that determine the equilibrium outcomes for R and b for $a = Rb$.

Figure 6 plots the supply curves for (b, R) and (a, R) defined in (1) and (4), respectively, for the normal distribution. It also plots the demand curves defined in (6) and (3) for the logarithmic utility function. With the timing assumed so far, whether the borrower chooses debt net or gross of interest is irrelevant.

3 Timing of moves and multiplicity: Related literature

The timing of moves assumed above, with the creditors moving first, amounts to assuming that the borrower in this two-period game takes the current price of debt as given.[13] The more common assumption in the literature is that the borrower moves first, choosing debt levels b or a, and facing a schedule of interest rates as a function of those levels of debt, $R = R\left(b\right)$ or $R = \frac{1}{q(a)}$, depending on whether the choice is b or a, respectively.

Suppose the schedule the borrower faces is $q\left(a\right)$, corresponding to the supply curve derived from (4) and depicted in the right-hand panel of Figure 6. This is a monoton-

[13] In the dynamic game, the contemporaneous price is taken as given, but this is not true for the future prices.

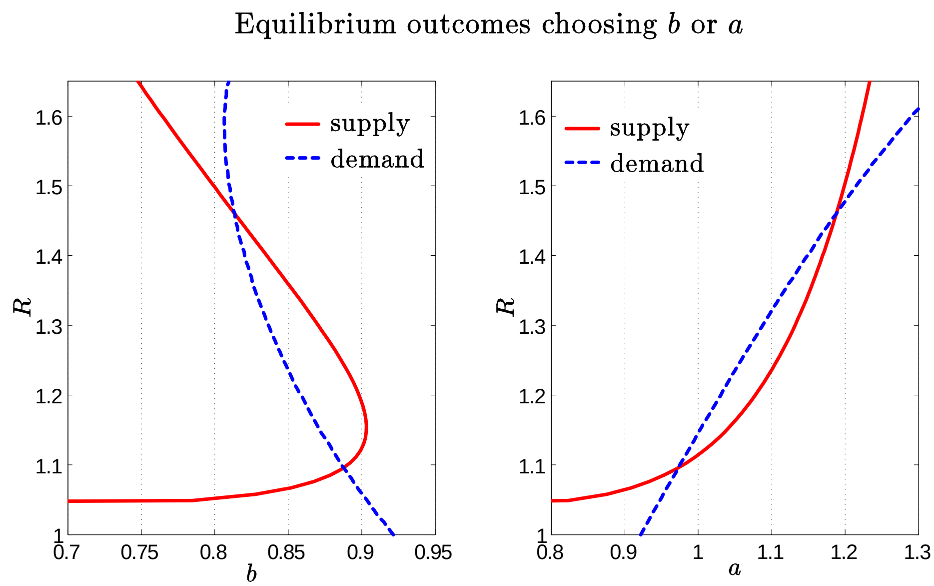

Figure 6: Choosing value of debt at maturity a or amount borrowed b

ically increasing function. Since the borrower can choose a, the borrower is always going to choose in the low R/low a part of the schedule. The borrower is also going to take into account the monopoly power in choosing the level of a. These are the assumptions in Aguiar and Gopinath (2006) and Arellano (2008). The equilibrium is unique.

Suppose now that the borrower faces the full supply curve as depicted in Figure 2 with an increasing low rate schedule and a decreasing high rate schedule. Then by picking b, the borrower is not able to select the equilibrium outcome.[14] There are multiple possible interest rates that make creditors equally happy. The way this can be formalized, as in Calvo (1988),[15] is with multiple interest rate functions $R(b)$, which can be the low rate increasing schedule or the high rate decreasing one. Any other combination of those two schedules is also possible. The borrower is offered one schedule of the interest rate as a function of the debt level b and chooses debt optimally given the schedule.

In summary, the assumption on the timing of moves is a key assumption to have

[14]Trivially, it is still possible to obtain uniqueness in the case in which the borrower faces the supply curve in R and b defined by (1). If the borrower picks R, then it is able to select the low rate equilibrium directly. That is essentially what happens when the borrower faces the schedule $R(a)$ and picks a.

[15]In Calvo (1988), debt is exogenous.

multiple equilibria or a single equilibrium. If the creditors move first, there are multiple equilibrium interest rates and debt levels, and they are the same equilibria whether the borrower chooses current debt or debt at maturity. Instead, if the borrower moves first and chooses debt at maturity, as in Aguiar and Gopinath (2006) and Arellano (2008), there is a single equilibrium. Choosing debt at maturity amounts to picking the probability of default and therefore the interest rate as well. Finally, if the borrower moves first and chooses the current level of debt, given an interest rate schedule defined as a one-to-one mapping from b to R, then the equilibrium will depend on the schedule and there is a continuum of equilibrium schedules. This is the approach in Calvo (1988). It is also the approach that we will follow in the dynamic computations in the next section.

Lorenzoni and Werning (2013) Lorenzoni and Werning (2013) use a dynamic, simplified version of the Calvo (1988) model, in which the borrower is a government with exogenous deficits or surpluses. In a two-period version, there is an exogenous deficit in the first period $-s^h$, with $s^h > 0$. In the second period, the surplus is stochastic, $s \in \left[-s^h, S\right]$, with density $f(s)$ and corresponding cdf $F(s)$. In order to finance the deficit in the first period, the government needs to borrow $b = s^h$. In the second period, it is possible to pay back the debt if $s \geq bR$, where R is the gross interest rate charged by foreign lenders.

The creditors are competitive, they must make zero profits. It follows that $R^* = R\left(1 - F\left(bR\right)\right)$. If we had written $q = \frac{1}{R}$ and $a = bR$, the condition would be $R^* = \frac{1}{q}\left(1 - F\left(a\right)\right)$. As before, it is possible to use these equations to obtain functions $R\left(b\right)$ using the first equation and $q\left(a\right)$ using the second equation. These would be the two classes of schedules that were identified in the analysis earlier, when the government moves first. For the normal distribution, the schedules $R\left(b\right)$ and $q\left(a\right)$ will look like the supply curves in Figure 6. There are multiple equilibrium schedules $R\left(b\right)$. There is the good, increasing schedule and the bad, decreasing schedule, and there is a continuum of other schedules with points from any of those two schedules. The government that borrows $b = s^h$ may have to pay high or low $a = R\left(b\right)b$ depending on which schedule is being used with the corresponding probabilities of default.

What if the schedule, instead, is $q\left(a\right)$? The schedule is unique, but there are multiple points in the schedule that finance b. The government that borrows $q\left(a\right)a = s^h$ can do so with low a and low $\frac{1}{q}$ or with high a and high $\frac{1}{q}$. If the government is able to pick a, then implicitly it is picking the interest rate. Lorenzoni and Werning (2013) use an interesting argument for the inability of the government to pick the debt level a. For that they devise a game in which they divide the period into an infinite number of subperiods and do not allow for commitment in reissuing debt within the period. In that model, the government takes the price as given. The intuition is similar to the durable good monopoly result. In our model, the large agent also takes the price as given because of the timing assumption.

Even if there are multiple equilibria, with high and low interest rates, the high interest rate equilibria that Lorenzoni and Werning (2013) focus on are of a different type. They assume that debt is long term and characterize high rate equilibria with debt dilution. Because we assume debt is only short term, the model we analyze does not exhibit those equilibria.

Eaton-Gersovitz (1981) In the model in Eaton and Gersovitz (1981), the borrower moves first, so it is key whether the equilibrium schedule is in b or a. In our notation, they consider a schedule for $R(b)$. To be more precise, they assume that $a = \overline{R}(b)$, where $\overline{R}(b) = R(b)\,b$. Their equation (8) can be written using our notation as $\left[1 - \lambda\left(\overline{R}(b)\right)\right]\overline{R}(b) = R^* b$, where λ is the probability of default that depends on the level of debt at maturity. This is equivalent to

$$[1 - \lambda(R(b)\,b)]\,R(b) = R^*,$$

which is analogous to equation (1) in our model. As seen earlier, there are multiple schedules in this case.

For the case of the uniform or normal distributions, there is both an increasing and a decreasing schedule $R(b)$. In that case, $\overline{R}(b) = R(b)\,b$ first goes up with b and then goes down. Eaton and Gersovitz dismiss the decreasing schedule by assuming that $R(b)\,b$ cannot go down when b goes up. This amounts to excluding decreasing schedules by assumption.[16]

[16] See proof of Theorem 3 in Eaton and Gersovitz (1981).

4 The infinite period model: Numerical exploration

In order to keep the analysis closer to the literature that has computed equilibria with sovereign debt crises in models without a role for sunspots, as in Aguiar and Gopinath (2006) and Arellano (2008), we consider their timing in which the borrower moves first. In order for there to be a role for sunspots, the borrower chooses the current debt rather than debt at maturity.[17]

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. The endowment y follows a Markov process with conditional distribution $F(y'|y)$. At the beginning of every period, after observing the endowment realization y, the borrower can decide whether to repay the debt or to default. Upon default, the borrower is permanently excluded from financial markets and the value of the endowment becomes $y^d \in \mathbb{R}_+$ forever.[18]

The period utility function, $U(c)$, is assumed to be strictly increasing and strictly concave and to satisfy standard Inada conditions. Thus,

$$V^{aut} = \frac{U(y^d)}{1-\beta} \tag{7}$$

is the value of default.

We allow for a sunspot variable s that takes values in $S = \{1, 2, ..., N\}$ and follows a Markov distribution $p(s'|s)$. Upon not having defaulted in the past, every period the borrower chooses the current debt b' given an interest rate schedule that may depend on the realization of the sunspot variable s.

The case with two schedules We analyze the case with two possible schedules for the bimodal distribution studied earlier. The sunspot variable can take two possible realizations, $s = 1, 2$, which indexes the interest rate schedule $R(b', y, s)$ faced by the borrower. The value for the borrower that did not default is given by $V(\omega, y, s)$,

[17] The computations of the alternative timing, where the lenders move first so the borrower takes the price as given, is harder because of the discontinuty of the demand function discussed in Section 2.1.

[18] Note that the value of autarky is independent of the state previous to default. This substantially simplifies the analysis.

satisfying

$$
\begin{aligned}
V(\omega, y, s) &= \max_{c, b', \omega'} \left\{ U(c) + \beta \mathbb{E}_{y', s'} \left[\max \left\{ V^{aut}, V(\omega', y', s;) \right\} | y, s \right] \right\} \\
& \quad \text{subject to} \\
c &\leq \omega + b' \\
\omega' &= y' - b' R(b', y, s) \\
b' &\leq \underline{b}
\end{aligned} \tag{8}
$$

Wealth ω is used as a state variable (instead of current debt) because it reduces the dimensionality of the state space.[19] The borrowing limit is important. Since the borrower receives a unit of consumption for every unit of debt issued, it could always postpone default by issuing more debt. This is ruled out by imposing a maximum amount of debt.

The interest rate schedule $R(b', y, s)$ is a function of the amount of debt because default probabilities depend on it, and the interest rate reflects the likelihood of default. It is also a function of current output; since the endowment follows a Markov process, it contains information about future default probabilities. Naturally, there are infinitely many possible pairs of schedules. We focus only on the pair in which, given one possible value of the sunspot, the schedules either always pick the low interest rate or always the high interest rate.

Default follows a threshold $\underline{y}(b', y, s, s')$ such that the optimal rule is to pay the debt as long as $y' \geq \underline{y}(b', y, s, s')$ and default otherwise.[20] The threshold for default is the level of $y' = \underline{y}(b', y, s, s')$ that solves

$$
V^{aut} = V\left(\omega', y', s'\right) = V\left(y' - b' R(b', y, s), y', s'\right). \tag{9}
$$

Creditors offer their amount of funds as long as the expected return is R^*. The arbitrage condition for the risk-free creditors that pins down the schedule $R(b', y, s)$ is

$$
R^* = R(b', y, s) \sum_{s'=1,2} p(s'|s) \left[1 - F\left(\underline{y}(b', y, s, s') | y \right) \right], \tag{10}
$$

[19] If we were to keep current debt b as a state, we would also need to know the previous period interest rate that is a function of the debt level in the previous period.

[20] All equilibria have this property as long as $\frac{\partial V(\omega, y, s)}{\partial y} \geq 0$, which is the case with non-negative serial correlation of the endowment process.

where $\underline{y}(b', y, s, s')$ is defined by (9).

Equilibrium An equilibrium is given by functions

$$V(\omega, y, s), c(\omega, y, s), b'(\omega, y, s), R(b', y, s), \underline{y}(b', y, s, s')$$

such that

1. given $R(b', y, s)$, policies $c(\omega, y, s)$ and $b'(\omega, y, s)$ solve (8) and achieve $V(\omega, y, s)$.

2. given $V(\omega, y, s)$, the default threshold $\underline{y}(b', y, s, s')$ solves (9).

3. the schedule $R(b', y, s)$ satisfies condition (10).

4.1 Simulations

In this section, we compute equilibria to show that the model can replicate salient features of the recent European sovereign debt crisis. In particular, we discuss to what extent the European spread data can be generated by a model of this type. The discussion includes the effects of policy interventions that resemble the OMT program announced by the ECB in 2012.

Parameter values As discussed earlier, the key parameters to generate multiplicity are the ones that govern the stochastic process for the endowment, which must alternate from being relatively high to being relatively low. We interpret the low endowment regime as the possibility of relatively long periods of stagnation, whereas the high endowment regime is associated with periods of relatively high growth. Our interpretation is motivated by the ongoing debate regarding secular stagnation in Europe, which is consistent with the alternating regimes of growth rates documented by Hamilton (1989) and with the evidence provided in Jones and Olken (2008).

We now turn to specifics. We construct a bimodal distribution made out of two normals. Every period, with probability π, the endowment is drawn from $N(\mu_1, \sigma)$, whereas with probability $1 - \pi$, the endowment is drawn from $N(\mu_2, \sigma)$ with $\mu_1 < \mu_2$. For simplicity, we assume both distributions have the same standard deviation. The relatively large differences between the means of the two distributions, required to exhibit multiplicity, are interpreted as the effect of different growth rates of output for a prolonged period of time. A period in the model is several years, such as a decade. This period is similar to the average maturity of debt for most of the European countries under discussion, so it is consistent with a single period maturity in the model. To calibrate the difference between the means, we estimate a Markov-switching regime for the growth rate of output for Portugal, Spain, and Italy, and as well as for Argentina and the United States, using data from 1960 to 2014. The difference in yearly growth rates between the high and low regimes is 4.77% for Spain, 5.11% for Portugal, and 3.45% for Italy.[21] Because of the high convergence of these three economies during the 1960s, we also estimated the system starting in 1970. The difference between the means drops to 3.5% for Spain, 4.85% for Portugal, and 3.14% for Italy. A growth rate differential between the high growth regime and the low growth regime of between 3.5% and 5% delivers an income gap between 40% and 60% in 10 years. Thus, we assume in our benchmark case that $\mu_2 = 1.5\mu_1$.[22]

In addition, we set the probability of drawing a value for the endowment from the low distribution to be $\pi = 0.3$. Computing the unconditional probabilities from our estimates, one obtains a value of 0.31 for Italy, 0.40 for Spain, and 0.53 for Portugal using the estimation starting in 1970.[23] We will report results using higher values for π.

Finally, note that we assumed the probability π to be independent of the state. We do this because in spite of the clear evidence of persistency in our estimates using a yearly date, is that we interpret the period as a decade, so an *i.i.d.* distribution therefore appears more natural.

The model has a few additional parameters. The first three are not controversial. First, we set the international interest rate $R^* = 1.20$, roughly consistent with a 2% yearly rate during a decade. We allow the discount factor in preferences for the borrower to be higher, so $\beta = 0.7$, consistent with a yearly discount factor of 0.96. Preferences exhibit a constant relative risk aversion with parameter $\gamma = 6$, so as to have a relatively strong preference for consumption smoothing.

[21]The corresponding numbers for Argentina and the United States are 8.78% and 3.45%. The probability of the low growth state in the United States is lower than for the other countries. The results do not change if we use data starting in 1970 in either case. See Appendix 3.

[22]If we also allow for standard deviations that depend on the state, the results barely change for Spain and Portugal. However, for Italy, the estimates in this case present no difference in the means, but the standard deviation in one of the states becomes very high.

[23]The probabilities of the bad state are higher—between 45% and 65% if we estimate the model using data starting in 1960.

There are two remaining parameters: the value for consumption following default and the probability of the sunspot that coordinates on alternative schedules. Following default, the borrower is cut off from international credit markets. To the extent that integration to world markets is associated with the possibility of rapid growth, it is natural to think that default could substantially reduce the probability of drawing from the high-endowment distribution. Following this notion, we set the value of endowment following default to be equal to $y^d = \mu_1 = 4$.

Finally, we assume that the sunspot distribution is *i.i.d.* $p(s'|s) = p$, and we set the probability of the sunspot that coordinates on the high interest rate schedule to be 0.2. All the results we show are essentially the same if we set that probability to be 0.4.

Characterization of equilibria Figure 7 plots the schedule of yearly interest rates as a function of the debt level. For debt levels between 1.8 and 2.2, there are two possible interest rates. Note that when there is multiplicity, rates range from 1.8% per year to 5.6%, so this example delivers a spread of about 3.8% a year, which is close to the maximum value of Spanish and Italian spreads but much smaller than the ones of Portugal, Ireland, or Greece. This depends on our choice of a key parameter: the probability of entering a period of stagnation, $\pi = 0.3$. If we set $\pi = 0.5$, the model generates a spread of 9.5%. This number is still lower than those spreads observed for Portugal, Ireland, or Greece. One reason for the observed high spreads in those countries could be a run-up to default—which has it already happened in Greece—that our long-run calibration cannot capture.

A particular feature of the increasing schedule is the apparently flat sections.[24] This is the result of having two normal distributions with relatively large differences in mean, and very small standard deviations. Note that the "good" distribution has most of the mass between 5.8 and 6.2 so that if the threshold is below 5.8 but not too far away, increases in the threshold have a negligible effect on the probability of default, so they barely affect the interest rates.

Figure 8 plots the policy functions for the debt levels as a function of wealth for different realizations of the sunspot. The dashed red (solid blue) line corresponds to the case in which the sunspot selects, for each value of the debt, the high (low) interest rate. The horizontal axis is the wealth at the beginning of the period, which

[24] The schedules are not exactly flat.

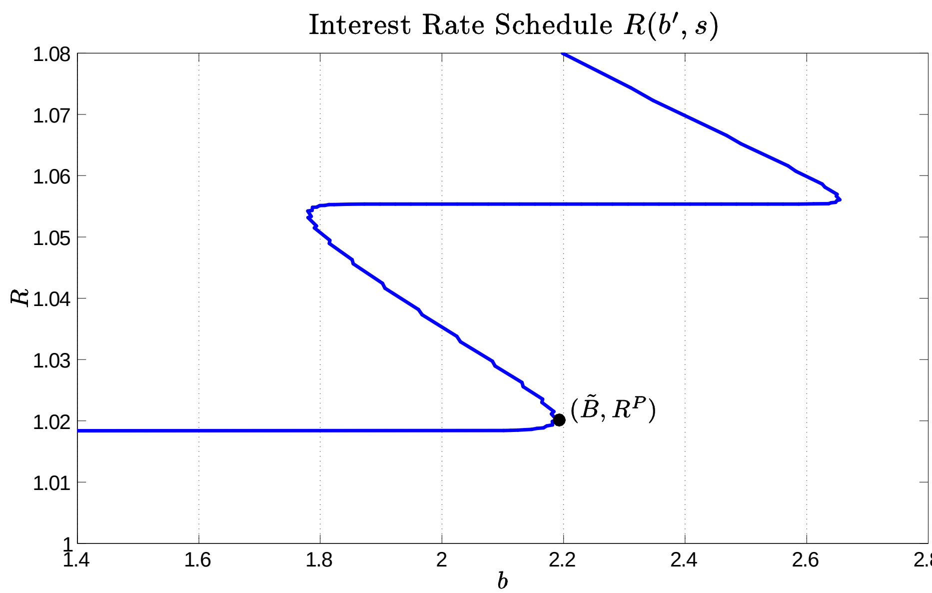

Figure 7: Interest rate schedule $R(b', s)$

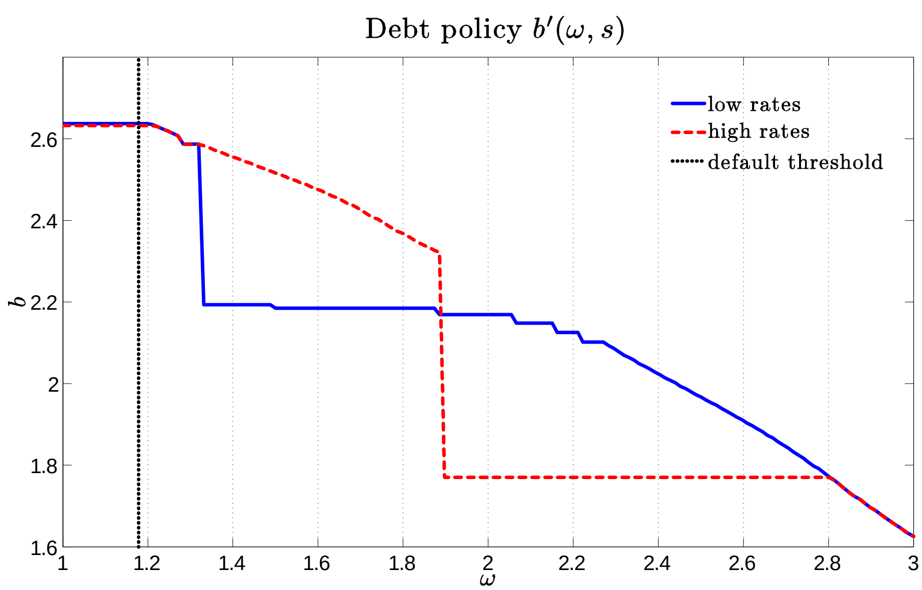

Figure 8: Debt policy function $b'(\omega, s)$

is equal to the realization of the endowment minus debt gross of interest payments. For values of the wealth above 2.8, the two policy functions coincide. This corresponds to choices of debt that are below the value beyond which there is multiplicity. Thus, for this region, the realization of the sunspot is inessential, and as wealth goes down, the amount borrowed goes up—the standard consumption-smoothing result. However, for values of wealth close to but lower than 2.8, the behavior critically depends on the realization of the sunspot. If the sunspot selects the lower schedule, debt keeps on increasing as wealth goes down. Instead, if the sunspot selects the high schedule, debt is invariant with wealth. The reason is that borrowing is close to 1.8, a value such that the interest rate schedule exhibits a discontinuous jump in the interest rate when the sunspot turns out bad (see Figure 7). Faced with such a high effect on the interest rate, the borrower reduces consumption one to one with wealth. Eventually, however, when wealth is sufficiently low—and so is current consumption—the borrower is willing to pay the fixed cost of the high interest rate on all the debt, at which point debt increases discontinuously. From there on, increases in the debt have very marginal effects on the rate, so debt once again goes up, one to one with the reduction in wealth.

The behavior of the policy function when the sunspot pins down the low interest rate schedule is similar, except that the effects occur for lower values of wealth: the policy function flattens when wealth reaches around 2.2 and jumps up discontinuously once wealth is around 1.9.

Figure 9 plots the equilibrium interest rates as a function of wealth for the two different realizations of the sunspot. As before, the dashed red (solid blue) line is the interest rate if the sunspot selects the high (low) interest rate schedule. As mentioned earlier, there are infinite ways to construct pairs of schedules; we just chose to focus on the ones that always choose either the low interest rate or the high interest rate.

It is interesting to highlight how the borrower's choices are key to understanding equilibrium outcomes. The region of multiplicity is roughly the one where wealth is between 1.3 and 2.8. And notice that, contrary to what one could have expected, the equilibrium interest rate when in the bad sunspot (high interest rate) is lower, even if by a very small amount, than the interest rate in the good sunspot for values of wealth between 1.9 and 2.8, close to two-thirds of the multiplicity region (see Figure 9). The reason is simple: it is precisely because the borrower faces the high interest rate schedule that it is willing to adjust consumption and avoid those high interest rates—a form of endogenous austerity. This rationalizes the notion that the probability of a

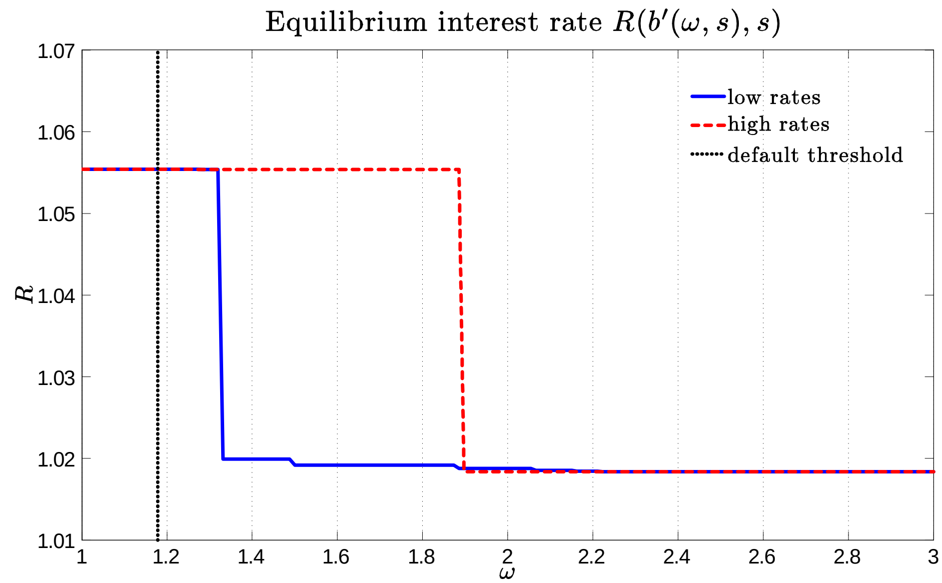

Figure 9: Equilibrium interest rates $R\left(b'\left(\omega,s\right),y,s\right)$

crisis may have a disciplinary effect. This effect, however, is present only up to a point: once wealth is below 1.9, the borrower, facing the high interest rate schedule, has such a pressing motive to borrow that he is willing to borrow at very high rates. When shocks bring the borrower to this region, debt levels and interest rates go up in the data—another feature of the data throughout the European sovereign debt crisis.

This endogeneity of debt implies that equilibrium interest rates are less revealing of the existence of multiplicity than borrowing choices. In Figure 10, we plot the ratio of optimal debt choices (left vertical axis) and the interest rate differential (right vertical axis) for the two values of the sunspot. We focus on the range of values of wealth for which there is multiplicity, roughly between 1.4 and 2.8. Although debt choices are very different for the whole range, the interest rate differentials are barely different for a large fraction of the range.

Policy intervention and multiplicity We now use the solution of the model to illustrate how a sovereign debt crisis can unfold, thereby shedding light on the role of the sunspot realization and the role of policy. The first step is to define the policy intervention. We assume there exists an institution with enough funds that can offer by itself an amount larger than the value $\widetilde{B}$, in Figure 7. A policy consists of a pair $(\widetilde{B}, R^P)$ such that the institution is willing to lend funds to the borrower at a rate R^P,

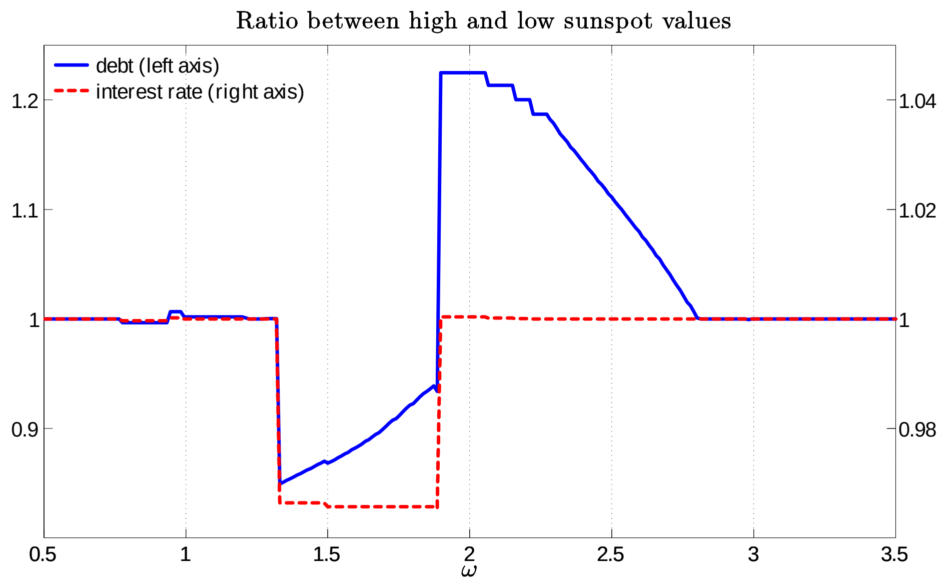

Figure 10: Debt and interest rate comparison

up to a maximum value of $\widetilde{B}$. Let $P^* = (\widetilde{B} + \delta, \widetilde{R} + \varepsilon)$ for low enough values of ε, δ where $\widetilde{R}$ is depicted in Figure 7 and $\varepsilon > 0, \delta > 0$.

Then, by the same logic as that explained in Section 2.2, policy P^* eliminates the high interest rate schedule. In addition, in equilibrium, the institution lends no funds. Note the importance of the maximum level $\widetilde{B}$: if δ is too large and ε small enough, it may be optimal to borrow from the institution amounts larger than $\widetilde{B}$. But at those values, the institution's expected return is lower than R^*.

Policy P^*, by removing the high equilibrium schedule, is equivalent to setting the probability of the sunspot to zero. To put it differently, assume that the institution implements policy P^* with probability ϕ and implements no policy with probability $(1 - \phi)$. This is equivalent to an economy without any policy intervention and the probability of the bad sunspot being $p(1-\phi)$. By reinterpreting the parameters above, we can simulate a sovereign default crisis and the policy intervention: we let $\widetilde{p} = 0.4$ and $\phi = 0.5$, so $\widetilde{p}(1 - \phi) = p = 0.2$.

In Figure 11, we plot the time series of interest rates and debt choices after a sequence of shocks. We start the economy with wealth equal to 3.2, a value for which there is no multiplicity, and assume that the endowment shock is equal to four every period. We assume that the good sunspot realizes for four periods, after which time the

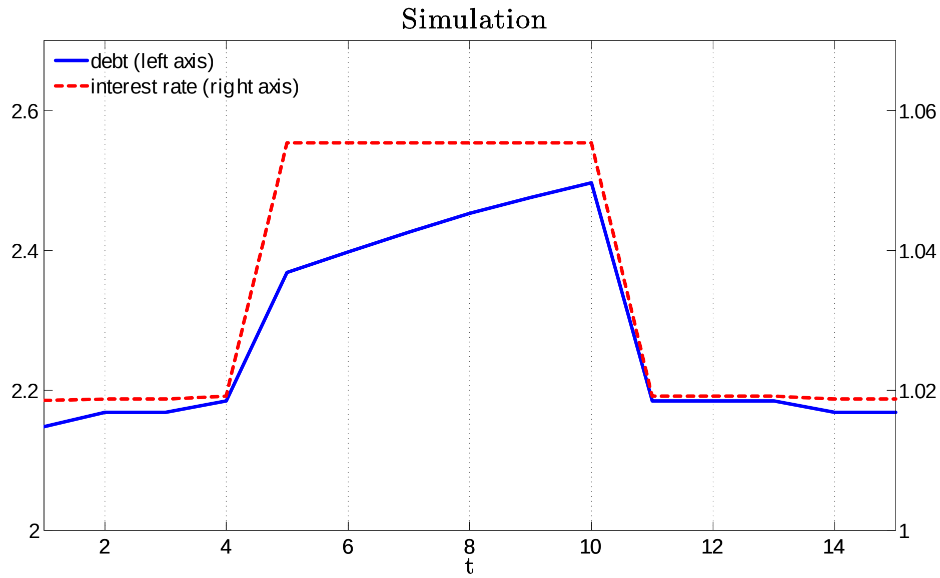

Figure 11: Simulation

bad sunspot realizes every period. Nature chooses that policy is only implemented at period $t = 11$ and remains in place thereafter. As can be seen from Figure 11, spreads go up once the bad sunspot is realized and come down once policy is implemented. Note also that debt goes up when the spreads go up, and then it comes down when the spreads come down, induced by the policy. In this way, austerity arises endogenously once the policy is implemented.[25]

5 Concluding remarks

In models with sovereign debt, interest rates are high when default probabilities are high. The object of this paper is to investigate conditions under which the reverse is also true, that default probabilities are high because interest rates are high. This means that there can be equilibrium outcomes in which interest rates are unnecessarily high and in which policy arrangements can bring them down. This exploration is motivated by the recent sovereign debt crisis in Europe, but it is also motivated by a literature

[25]It is worth emphasizing that we chose to simulate the case in which the borrower moves first and faces a schedule in which the interest rate depends on current actions. Had we solved the case in which the lenders move first so that the borrower takes today's intrest rates as given, we conjecture that this endogenous austerity would not be as strong.

that does not seem to be consensual in this respect. Indeed, although Eaton and Gersovitz (1981) claim that there is a single equilibrium, Calvo (1988), using a similar structure, shows that there are both high and low interest rate equilibrium schedules. Aguiar and Gopinath (2006) and Arellano (2008), building on Eaton and Gersovitz, modify an important assumption on the choice of debt by the large player and find a single equilibrium. We show that small changes in timing assumptions and actions of agents, which cannot be directly disciplined by empirical evidence, can explain these conflicting results.

Assumptions on whether the country chooses the debt net of interest payments or gross of those payments, or whether the borrower moves first or the creditors do, are not assumptions that can be obtained directly from empirical evidence. But there is indirect evidence. The multiplicity of equilibria that arises under some of those assumptions is consistent with the large and abrupt movements in interest rates that are observed in sovereign debt crises, whereas the single equilibrium is not.

We also simulate a dynamic version of the model in which a sunspot variable can induce high frequency movements in interest rate equilibria. We believe this can be a reading of a sovereign debt crisis. If so, then policies of large purchases of sovereign debt, at penalty rates, in the spirit of the ones announced by the ECB back in 2012, can have the effect that they seem to have had, of bringing down sovereign debt spreads.

According to this view, the ruling of the German constitutional court in early 2014, which "found that the central bank had overstepped its mandate and that OMT was a backdoor to 'monetary financing' of governments outlawed under European treaties," is unfounded.[26] This conclusion is not necessarily warranted: alternative interpretations based on the concept that the OMT actually implies future transfers to countries experiencing high spreads from countries not experiencing them, which will eventually be used to pay for the debt, may also be consistent with the data.

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Appendix 1

In this appendix, we perturb the game described in Section 2 by introducing a second stage at the end of period 1 that allows for partial renegotiation. We then explore the robustness of the equilibria described in Section 2 to "very small" perturbations, in a sense we make precise later on.

Specifically, and given any outcome (b, R) in the first stage of period 1, nature allows the borrower, with probability $\pi \in (0, 1)$, to make a take-it-or-leave-it offer of an alternative interest rate equal to $R - \delta$ to a coalition of a fraction α of lenders where $\delta > 0$ is exogenously given.[27] The coalition then chooses to accept the new rate or to keep the one in the first stage. Period 2 is the same as before: given the amount owed by the borrower and the realization of the endowment shock, the borrower decides to default or pay the debt in full. The payoff following default is as before.

We denote by $\alpha \in (0, 1)$ the measure of lenders that may be called into the coalition. It is important to emphasize that in this second stage, it is the coalition rather than each individual lender making decisions. Each agent in the coalition is treated equally. It is this assumption that imposes an α-limited degree of coordination. As $\alpha \to 0$, there is no degree of coordination, and as we will show, the refinement requires α to be strictly bounded above zero. The first stage is exactly as before: all lenders—the α ones that can be called into the coalition and $(1 - \alpha)$ who cannot—then compete among each other, so they all charge the same rate in the first stage.[28]

Let this perturbed game be denoted by $G^{\alpha}(\delta, \pi)$.[29] We first characterize equilibria in the games $G^{\alpha}(\pi, \delta)$. In the spirit of trembling-hand perfection, we explore, given α, which of the equilibria described in Section 2 are the limit of the sequence of equilibria of the games $G^{\alpha}(\pi, \delta)$ when $\pi \to 0, \delta \to 0$.

[27] Considering only reductions in interest rates is without loss of generality. If the borrower had the option of choosing higher interest rates, he would never do so.

[28] Note that the perturbation introduces ex ante heterogeneity. We will focus on the limiting cases where $\delta \to 0$ and $\pi \mapsto 0$, so the heterogeneity is vanishing in the limit.

[29] The original game is equivalent to $G^{\alpha}(\delta, 0)$ or $G^{\alpha}(0, \pi)$.

Definition 2 *Given $\alpha \in (0,1)$, an equilibrium (R,b) in the game $G^{\alpha}(0,0)$ is robust to an $\alpha-$degree of coordination if it is the limit of the sequence of equilibria in the games $G^{\alpha}(\delta,\pi)$ when $\delta \to 0$ and $\pi \to 0$.*

We will show that the equilibria in the decreasing part of the zero-profit schedule

$$R^{*} \equiv R(b)\left[1 - F\left(1 + bR(b)\right)\right] \tag{11}$$

do not survive a refinement based on this perturbation, whereas equilibria in the increasing part do. Two assumptions are key to obtaining the results. First, the auction in the first stage must be anonymous (Lemma 1).[30] Second, a strictly positive degree of coordination of lenders is required (Result 2).

We prove the results in a series of steps. First, we show that, as long as the auction in the first stage is anonymous, there is no equilibrium in the perturbed game in which the offer is accepted.

Lemma 1 *In any equilibrium of the perturbed game in which nature allows the borrower to make the offer $R-\delta$, the offer is rejected by the coalition if the auction in the first stage of the game is anonymous.*

Proof. Assume there is one equilibria with $\delta > 0$ where the offer is accepted. The $\alpha-$members of the coalition get an interest rate of R^{c} with probability π and an interest rate of $R^{c} - \delta$ with probability $(1-\pi)$, whereas the lenders that cannot be part of the coalition get an interest rate of R^{n}. The expected return for lenders within the coalition is

$$\begin{aligned} R^{*} &= (1-\pi)R^{c}[1 - F(1 + b\left[\alpha R^{c} + (1-\alpha)R^{n}\right])] \\ &\quad + \pi(R^{c}-\delta)[1 - F(1 + b[\alpha R^{c} + (1-\alpha)R^{n} - \alpha\delta])], \end{aligned} \tag{12}$$

whereas the condition for the $(1-\alpha)$ fraction of agents that do not get to be part of the coalition is

$$\begin{aligned} R^{*} &= (1-\pi)R^{n}[1 - F(1 + b\left[\alpha R^{c} + (1-\alpha)R^{n}\right])] \\ &\quad + \pi R^{n}[1 - F(1 + b[\alpha R^{c} + (1-\alpha)R^{n} - \alpha\delta])]. \end{aligned} \tag{13}$$

[30] Anonymity is irrelevant for the game in Section 2, where all agents are homogeneous. But the perturbation introduces heterogeneity, so anonymity is important in the perturbed game.

As $\delta > 0$, $R^n < R^c$. It is immediate that this cannot be an equilibrium in a second price auction, where $R^n = R^c$. Consider now a first price auction, where each lender receives the interest rate offered. Note that for the borrower, borrowing from each lender type implies the same expected payment. However, the lenders not in the coalition imply a payment with certainty, so the borrower will choose those lenders first. Thus, a fraction $(1 - \alpha)$ receives an interest rate of R^n and the fraction α receives the interest rate $R^c > R^n$. It immediately follows that in an anonymous auction, the noncoalition lenders best interest is to offer their funds at the rate R^c, where profits are higher than R^*. ■

If the auction is not anonymous, the borrower can fully discriminate in a first price option and conditions (12) and (13) fully characterize the equilibrium interest rates. Once the borrower cannot discriminate the lender's type, the second price auction provides incentives for truthful revelation but implies a unique interest rate, which breaks down the equilibrium. On the other hand, in a first price auction, agents not in the coalition do not have incentives to reveal their type, which also breakes the proposed equilibrium.

Note also that the perturbation we consider is a simple one in which agents know ex ante if they belong in the potential coalition or not. A more general perturbation would allow for each lender to have a probability $\alpha(j)$ of belonging to the coalition, with $\int_0^b \alpha(j)dj = \alpha$. The only case in which there can be an equilibrium with an anonymous auction in which an offer is accepted is the knife-edge case in which $\alpha(j) = \alpha$ for all j, so (13) is not an equilibrium condition anymore. In this case all agents are ex ante identical and anonymity plays no role.

We now characterize the conditions under which an offer will be accepted by the coalition for δ small enough.

Let the outcome of the first stage be a point in the schedule (R, b), defined by (11). Assume that nature allows the borrower to offer $R - \delta$ to the coalition. If it accepts the new rate, their return will be given by

$$(R - \delta) \left[1 - F\left(1 + (1 - \alpha)bR + \alpha b\left(R - \delta\right)\right)\right] \equiv E(\delta).$$

The new rate reduces the payment in case of no default, but it reduces the probability of default, so the net effect depends on which effect dominates. We now find a condition such that the second effect dominates, so reductions in the interest rates (positive values for δ) can increase the expected payment to the coalition for small enough perturbations, so δ is close to zero. For that, we derive the expression above $E(\delta)$ with respect to δ and evaluate it at $\delta = 0$ to obtain

$$E'(\delta = 0) = -\left[1 - F\left(1 + b\right)\right] + R[f\left(1 + bR\right)\alpha b].$$

This expression is positive when

$$Rb\frac{f\left(1 + bR\right)}{\left[1 - F\left(1 + b\right)\right]} \equiv H(R, b) > \frac{1}{\alpha},$$

in which case the coalition would accept the offer for δ small enough.[31] We now show the results mentioned above.

Result 1: *If the pair (R_1, b_1) is an equilibrium in the original game and is in the increasing part of the schedule defined by (11), then it is robust to an $\alpha-$degree of coordination for any α.*

Proof. Differencing the identity (11) with respect to b, we obtain

$$R^{'}(b) = \frac{R^2 f(1 + bR)}{[[1 - F(1 + bR)] - Rbf(1 + bR)]}.$$

If (R_1, b_1) is in the increasing part of the schedule, $R^{'}(b_1) > 0$. Since the numerator is positive, this implies that the denominator must be negative, which implies that

$$R_1 b_1 \frac{f(1 + b_1 R_1)}{[1 - F(1 + b_1 R_1)]} \equiv H(R_1, b_1) < 1.$$

Thus, $H(R_1, b_1) < 1 < \frac{1}{\alpha}$ for any $\alpha \in (0, 1]$, which means that the offer is not accepted for any degree of coordination, for some δ_1 that is small enough. This means that the equilibrium in the original game, (R_1, b_1), is also an equilibrium in the perturbed game, $G^\alpha(\pi, \delta)$, for any $\alpha > 0$, any π, and any $\delta < \delta_{1.}$ It therefore follows that (R_1, b_1) is the limit of this sequence of games when $\delta \to 0, \pi \to 0$. ■

Result 2: *If the pair (R_2, b_2) is an equilibrium in the original game and is in the decreasing part of the schedule defined by (11), then it is not robust to an $\alpha-$degree*

[31] Note that the smaller the coalition, the strongest is this condition. This is why the coalition is important.

of coordination for any $\alpha > \alpha^{\min}$, where $\alpha^{\min} < 1$ is the minimal required degree of coordination.

Proof. If (R_2, b_2) is in the decreasing part of the schedule, $R'(b_2) < 0$. Since the numerator is positive,

$$R_2 b_2 \frac{f(1 + b_2 R_2)}{[1 - F(1 + b_2 R_2)]} \equiv H(R_2, b_2) > 1.$$

Let

$$\alpha^{\min} = \frac{1}{H(R_2, b_2)} < 1.$$

Assume that (R_2, b_2) is robust to a $\alpha^{\min}-$ degree of coordination. This means that there is an equilibrium arbitrarily close to (R_2, b_2) in the game $G^{\alpha}(\pi, \delta)$ for $\alpha > \alpha^{\min}$ and small enough values for π and δ. By continuity of the function $H(R, b)$, it follows that if nature lets the borrower make an offer $R_2 - \delta$, it will be accepted by a coalition larger than $\alpha^{\min}$, which contradicts Lemma 1. ■

Note that the value of $\alpha^{\min}$ is related to the value of $H(R_2, b_2)$ relative to 1. When the slope of the schedule defined by (11) becomes very close to $-\infty$, which happens when the equilibria in the decreasing schedule get arbitrarily close to the equilibria in the increasing schedule, $H(R_2, b_2) \to 1$ and $\alpha^{\min} \to 1$, requiring an arbitrarily large degree of coordination.

Appendix 2

In this appendix, we show how small perturbations to the uniform distribution can give rise to multiple equilibria of the type obtained with the bimodal distribution.[32] Consider a perturbation $g(y)$ of the uniform distribution, so that the density would be $f(y) = \frac{1}{Y-1} + \gamma g(y)$, with $\int_1^Y g(y)dy = 0$. In particular, the function g can be $g(y) = \sin ky$, with $k = \frac{2\pi}{Y-1}N$, where N is a natural number. If $N = 0$, the distribution is uniform, so there is a single increasing schedule. If $N = 1$, there is a single full cycle added to the uniform distribution. The amplitude of the cycle (relative to the uniform distribution) is controlled by the parameter γ. The number of full cycles of the $\sin ky$ function added to the uniform is given by N. As $\gamma \to 0$, so does the perturbation.

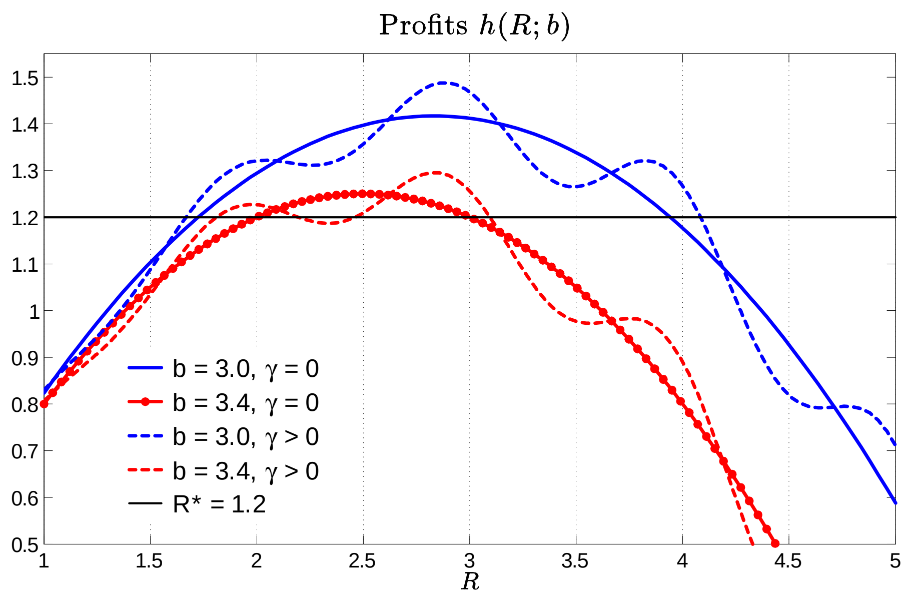

Figure 12: Perturbing the uniform distribution

Given a value for γ, the closer the debt to its maximum value, the larger the degree of multiplicity. The equation

$$\frac{1}{R} - \frac{1}{R^*}\left[1 - \frac{1+bR}{Y-1} - \gamma \sin kbR\right] = 0$$

has more than two solutions for R, for γ that can be made arbitrarily small, as long as b is close enough to $b^{\max}$. On the other hand, if b is lower than $b^{\max}$, there is always a

[32]The uniform distribution is used only as an example.

$\gamma > 0$ but small enough such that there are only two zeros to the function above.

An illustration is presented in Figure 12 for two levels of the debt and for two values of γ. As can be seen, when the debt is low, a positive value of γ is not enough to generate multiplicity, but multiplicity arises as the level of the debt goes up. Note that if γ is small, it may take a very long series to identify it in the data. Thus, it is hard to rule out this multiplicity based on calibrated versions of the distribution of output if the debt is close enough to its maximum.[33]

[33] This resembles the result in Cole and Kehoe (2000), where the fraction of short-term debt affects the chances of multiplicity.

Appendix 3: Bimodal Estimations

In this appendix, we explain the estimation procedure of the bimodal distribution for output used in the quantitative exercises of Section 4.

Let y_t denote GDP growth in period t. We assume that

$$y_t = \mu_t + \sigma \varepsilon_t, \tag{14}$$

where $\varepsilon \sim N(0,1)$ and $\mu_t = \mu(s_t)$. The random variable (regime) s_t follows an N-discrete Markov process with transition matrix P, where $P(j,i) = \Pr(s_{t+1} = j | s_t = i)$ for all $i, j = 1, 2, \ldots, N$. Let $\boldsymbol{\pi}$ be the stationary distribution of s_t, so that $\boldsymbol{\pi} = P\boldsymbol{\pi}$.

The density of y_t conditional on s_t is given by

$$f(y_t | s_t = j; \theta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{\left(y_t - \mu_j\right)^2}{2\sigma^2} \right\}, \tag{15}$$

where θ is a vector of parameters that contains $\left\{\mu_j, \sigma\right\}$ and the transition matrix P. Let η_t be an $N \times 1$ vector collecting conditional densities in (15) for all $j = 1, 2, \ldots, N$. Let $\xi_{t|\tau}$ be an $N \times 1$ vector collecting probabilities $\Pr(s_t = j | y^\tau; \theta)$, with $y^t = \{y_1, \ldots, y_t\}$ being the observed history of y_t up to time t.

It can be shown (see Hamilton 1994, chapter 22) that the log-likelihood of parameters θ, given an observed history y^T, is given by

$$\mathcal{L}(\theta) = \sum_{t=1}^{T} \log f\left(y_t | y^{t-1}; \theta\right) \tag{16}$$

with

$$f\left(y_t | y^{t-1}; \theta\right) = \mathbf{1}'\left(\xi_{t|t-1} \odot \eta_t\right),$$

where $\mathbf{1}$ is an $N \times 1$ vector of ones, and $\odot$ denotes element-by-element multiplication. The vector $\xi_{t|t-1}$ satisfies

$$\begin{aligned} \xi_{t|t} &= \frac{\xi_{t|t-1} \odot \eta_t}{\mathbf{1}'\left(\xi_{t|t-1} \odot \eta_t\right)} \\ \xi_{t+1|t} &= P\xi_{t|t}, \end{aligned} \tag{17}$$

which can be recursively computed given an initial condition $\xi_{1|0}$, which we set to be equal to $\boldsymbol{\pi}$.

For our estimations, we use annual GDP data from 1960-2014 for the United States, Spain, Portugal, Italy, and Argentina. We also consider the results using data from 1970-2014 only. Tables 1 and 2 show the results for the periods 1960-2014 and 1970-2014, respectively. Data for the United States, Spain, Portugal, and Italy come from the OECD database (http://stats.oecd.org/) and correspond to the series VPVOBARSA. For Argentina, we used the data set constructed by Buera, Navarro and Nicolini (2012). We assume that $N = 2$, and thus the growth rate μ can only take two values: μ_L and μ_H. The transition matrix P is determined by values p and q such that $p = \Pr\left(\mu_t = \mu_L | \mu_{t-1} = \mu_L\right)$ and $q = \Pr\left(\mu_t = \mu_H | \mu_{t-1} = \mu_H\right)$.

Table 1: Results for 1960-2014

	Parameters					Ergodic Distribution	
	μ_L	μ_H	σ	p	q	$\boldsymbol{\pi}(\mu_L)$	$\boldsymbol{\pi}(\mu_H)$
United States	-0.0022	0.0371	0.0145	0.33	0.86	0.17	0.83
Spain	0.0215	0.0692	0.0208	0.98	0.97	0.64	0.36
Portugal	0.0039	0.0548	0.0213	0.85	0.87	0.46	0.54
Italy	0.0123	0.0468	0.0209	0.98	0.97	0.58	0.42
Argentina	-0.0327	0.0551	0.0300	0.48	0.69	0.36	0.64

Table 2: Results for 1970-2014

	Parameters					Ergodic Distribution	
	μ_L	μ_H	σ	p	q	$\boldsymbol{\pi}(\mu_L)$	$\boldsymbol{\pi}(\mu_H)$
United States	-0.0041	0.0349	0.013	0.30	0.82	0.20	0.80
Spain	0.0042	0.0393	0.0158	0.84	0.89	0.40	0.60
Portugal	0.0035	0.052	0.022	0.84	0.81	0.53	0.47
Italy	-0.0071	0.0243	0.0200	0.95	0.98	0.31	0.69
Argentina	-0.038	0.0507	0.0300	0.47	0.66	0.39	0.61